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BUBBLE DYNAMICS IN A SUPERHEATED LIQUID

by

W. T. Sha and V. L. Shah

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ARGONNE NATIONAL LABORATORY, ARGONNE, ILLINOIS

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TABLE OF CONTENTS

	Page
NOMENCLATURE	8
ABSTRACT	9
I. INTRODUCTION	9
A. Bubble Dynamics	9
B. Present Work	9
II. GROWING AND COLLAPSING PROCESS	10
A. Vapor Bubbles	10
B. Equilibrium Conditions	10
C. Equilibrium States	11
D. Thermodynamic States on a Saturation Curve	12
E. Growing Process on Saturation Curve	14
1. Inertia-controlled Growth	15
2. Heat-transfer-controlled Growth	15
III. DYNAMICS OF A STATIONARY BUBBLE	16
A. Governing Equations	16
1. Conservation Equations for a Surrounding Liquid Phase	16
2. Interface Relations and Boundary Conditions at $r = R(t)$	17
B. Brief Review of Past Work	19
1. Inertia-controlled Growth	20
2. Heat-diffusion-controlled Growth	20
3. Inertia- and Heat-diffusion-controlled Growth	23
C. Growth of a Sodium-vapor Bubble	24
IV. MOTION OF A BUBBLE IN A VISCOUS LIQUID	26
A. Governing Equations	27
B. Brief Review of Some Theoretical Work	28

TABLE OF CONTENTS

		Page
V.	DYNAMICS OF A MOVING BUBBLE	32
	A. Governing Equations	32
	B. Past Work	33
	C. Order-of-magnitude Approach	35
	D. Present Work	37
	1. Stationary Medium	37
	2. Moving Medium	46
VI.	CONCLUDING REMARKS	47
ACKNOWLEDGMENTS		48
REFERENCES		49
BIBLIOGRAPHY		51

LIST OF FIGURES

No.	Title	Page
1.	Spherical Bubble Growing in a Superheated Liquid	11
2.	Saturation Curve for a Pure Substance	11
3.	Equilibrium States of Liquid and Vapor with a Curved Interface	12
4.	Growth and Collapse Regions for Sodium and Water-vapor Bubbles	14
5.	Equilibrium State for Inertia-controlled Growth	14
6.	Equilibrium State for Heat-transfer-controlled Growth	15
7.	Variation of Growth Rate in Superheated, Saturated, and Subcooled Regions	19
8.	Variation of Bubble Radius in Superheated, Saturated, and Subcooled Regions	20
9.	Effect of Pressure on Growth of a Sodium-vapor Bubble	25
10.	Effect of Superheat on Growth of a Sodium-vapor Bubble	25
11.	Comparison of Growth Rates of Water and Sodium-vapor Bubbles	26
12.	Terminal Velocity of a Sodium-vapor Bubble	31
13.	Drag Coefficient for a Sodium-vapor Bubble	31
14.	Comparison of Present Theory with Measurements of Florschuetz et al. for a Water-vapor Bubble	40
15.	Comparison of Present Theory with Measurements of Florschuetz et al. for an Ethanol-vapor Bubble	40
16.	Comparison of Present Theory with Measurements of Florschuetz et al. for an Isopropanol-vapor Bubble	41
17.	Effect of Jakob Number on Growth of a Sodium-vapor Bubble	41
18.	Effect of Jakob Number on Growth of a Water-vapor Bubble	42
19.	Effect of Jakob Number on Parameter $\overline{R}$ for a Sodium-vapor Bubble	42
20.	Effect of Jakob Number on Parameter $\overline{R}$ for a Water-vapor Bubble	43
21.	Effect of Jakob Number on Rising Velocity of a Bubble	44
22.	Growth of a Vapor Bubble in a Superheated Liquid	45
23.	Effect of Velocity on Growth of a Sodium-vapor Bubble	45

LIST OF TABLES

No.	Title	Page
I.	Results for Water Vapor with Two Values of Initial Velocity	39
II.	Results for Water Vapor with Two Values of Time Increment . . .	39

NOMENCLATURE

NOMENCLATURE

a	Constant in Eq. 4; coefficient in Eq. 104
b	Constant in Eq. 4; body forces; coefficient in Eq. 104; correction factor in Eq. 47
c	Specific heat; growth coefficient in Eq. 38
C_D	Drag coefficient
D	Drag force
e	Specific internal energy; surface viscosity coefficient in Eq. 71
E	Eötvös number, Eq. 58
F	Froude number, Eq. 57
g	Gravitational acceleration
Ja	Jakob number ($= \Delta T \rho c / L \rho_v$)
k	Thermal conductivity
L	Latent heat
M	Molecular weight; dimensionless parameter, Eq. 54
p	Pressure of surrounding liquid
p_v	Pressure of vapor
p_∞	Pressure of the surrounding liquid far away from bubble interface
$\dot{q}_b$	Average heat flux from boiling surface, Eq. 42
$\dot{q}'''$	Heat source
$\dot{Q}$	Heat-transfer rate
r	Radial coordinate
R^+	Dimensionless radius, Eq. 50
R	Radius of a bubble
$\dot{R}$	Growth rate
$\ddot{R}$	Rate of change of growth rate
$\mathcal{R}$	Universal gas constant
R_0	Initial radius
Re	Reynolds number, Eq. 55
t^+	Dimensionless time, Eq. 51
t	Time
t_w	Waiting time
T	Temperature of liquid
T_w	Wall temperature
T_v	Temperature of vapor
T_R	Temperature of liquid at bubble interface
T_∞	Temperature of liquid far away from bubble interface
u	Velocity of moving bubble
u_r	Radial velocity of liquid
u_ℓ	Velocity of a moving medium
u_{rel}	Relative velocity
u_R	Radial velocity of liquid at $r = R$
u_∞	Steady-state velocity of a rising bubble; velocity far from bubble interface
u_θ	Tangential velocity
$\bar{u}$	Velocity vector
v	Specific volume
W	Weber number, Eq. 56
Z	Axial distance traveled by a bubble
β	Second coefficient of viscosity; see Eq. 19
ρ'	Density of a bubble; droplet; sphere
ρ	Density of liquid
ρ_v	Density of vapor
σ	Surface tension
μ'	Viscosity of a bubble fluid or a droplet fluid
μ	Viscosity of liquid
α	Thermal diffusivity
θ	Angular coordinate; dimensionless temperature
τ_{rr}	Normal stress
$\tau_{r\theta}$	Tangential stress
ϵ	Defined in Eq. 29
$\dot{\epsilon}$	Rate of change of ϵ
ν	Kinematic viscosity
δ	Boundary-layer thickness
ΔT	Temperature difference; amount of superheat

Subscripts

R	At the surface of a sphere of radius R
v	Vapor
sat	Saturation value corresponding to a plane interface
init	Initial value
∞	Far away from bubble surface

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ABSTRACT

This report presents an extensive literature survey on bubble dynamics. Growth of a single spherical bubble moving in a uniformly superheated liquid is considered. Equations of motion and energy are presented in the forms that take into consideration the interaction between the motion and the growth. The fourth-order Runge-Kutta method is used to obtain a simultaneous solution of equations of motion and growth rate, and the solution is compared with available experimental results. Results for liquid sodium are presented for a range of pressures and Jakob numbers.

I. INTRODUCTION

A. Bubble Dynamics

Broadly defined, bubble dynamics deals with all phenomena associated with formation, growth, motion, and collapse of single or multiple bubbles. Since bubbles are encountered in a wide range of industrial equipment, bubble dynamics is the subject of study by mechanical engineers, chemical engineers, geologists, metallurgists, and researchers whose main interest is in boiling and cavitation. With the advent of nuclear reactors, more and more nuclear engineers are taking interest in solving problems related to bubble dynamics.

Although much work has been done in this field, the subject is still not completely understood. As a consequence of the extreme complexity of the interaction between a bubble and its surroundings, most of the design work is carried out on the basis of experimental trial-and-error approaches. However, theoretical modeling can play a significant role in providing physical insight and rendering equipment more efficient and reliable.

B. Present Work

The present study was initiated with a desire of understanding the dynamics of sodium vapor bubbles in a fast breeder reactor. After a brief review of past theoretical work relating to a stationary and moving bubble, a

solution is presented for a moving bubble that is simultaneously growing in a uniformly superheated liquid. The fourth-order Runge-Kutta method is used to obtain a simultaneous solution of equations of motion and growth rate, and the solution is compared with the available experimental results. Results for liquid sodium are presented for a range of pressures and Jakob numbers.

II. GROWING AND COLLAPSING PROCESS

A. Vapor Bubbles

A bubble nucleates in a superheated liquid due to vaporization of liquid molecules into a space that is unoccupied by liquid; e.g., a cavity on the heating wall or on a suspended foreign material. Once a bubble is formed, three factors control its further growth. These are mechanical forces, heat-transfer rates, and diffusion rates at the interface. Initially, the growth is controlled by inertia and surface-tension forces, but as the bubble size increases, the other factors become more and more important.

There are two kinds of vapor bubbles: cavitation and boiling. A cavitation bubble grows in a liquid that is at a low temperature and a negative pressure (i.e., pressure lower than a saturation pressure). The dominating factor controlling its growth is the inertia force. In this case, a solution of the equation of motion along with the continuity equation may be sufficient to determine the growth or collapse rate of a cavitation bubble.

A boiling bubble, on the other hand, grows in a liquid that is at a high (superheated) temperature. In this case, the heat-transfer rates at the interface control the growth rate, and the energy equation must be included to determine the bubble dynamics of a boiling bubble. Moreover, if a bubble contains gases in addition to pure vapor, the determination must include the diffusion equation.

The study in this report is restricted to the growth of a pure vapor boiling bubble.

B. Equilibrium Conditions

Consider a spherical vapor bubble of radius R, pressure p_V, and temperature T_V, which is in equilibrium with the surrounding liquid of temperature T and pressure p. (See Fig. 1.)

For thermodynamic equilibrium to exist at the curved interface, the following conditions have to be satisfied.

1. Both the vapor and liquid have to be in superheated states with respect to saturation temperatures corresponding to a plane interface; i.e.,

$$T > T_{sat}(p), \tag{1}$$

and

$$T_v > T_{sat}(p_v). \tag{2}$$

2. The vapor pressure p_v has to be greater than the liquid pressure p to balance the additional surface-tension force. The excess pressure in a bubble of radius R is given by the Gibbs equation,

$$p_v - p = \frac{2\sigma}{R}, \tag{3}$$

where σ is the surface tension.

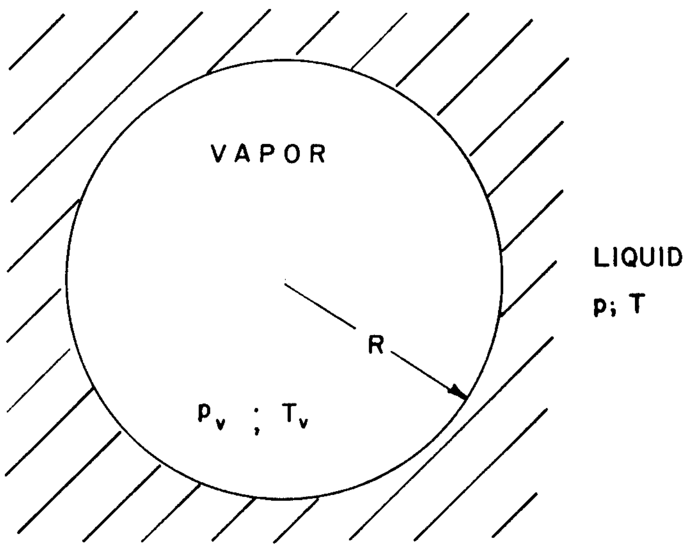

Fig. 1

Spherical Bubble Growing in a Superheated Liquid

C. Equilibrium States

Before describing the thermodynamic states of a growing bubble on a saturation curve, we shall consider the thermodynamic equilibrium states of a single-component fluid. Figure 2 shows a saturation curve on a pressure-volume diagram for a pure substance. The curve ABCDEF is an isotherm of temperature T. The isotherm can be approximated by the van der Waal equation,

$$\left[p + \frac{a}{(Mv)^2}\right](Mv - b) = \mathcal{R}T. \tag{4}$$

The curves AB and EF represent the thermodynamic states of a single-phase liquid

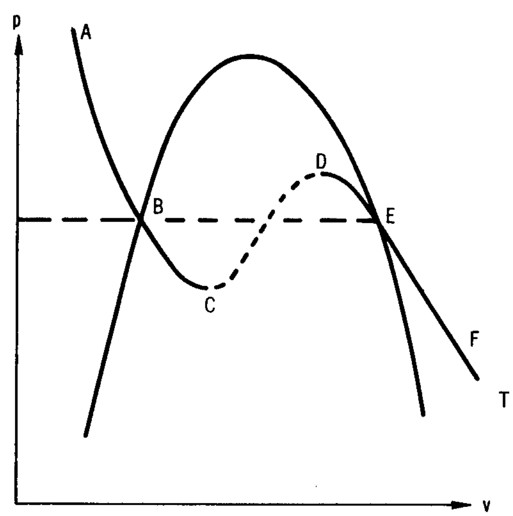

Fig. 2

Saturation Curve for a Pure Substance

and a single-phase vapor, respectively. When both liquid and vapor are in equilibrium with a plane interface, positions B and E represent the states of liquid and vapor, respectively. The curves BC and DE represent the unstable states of superheated liquid and subcooled vapor, respectively.

For a spherical bubble, the interface is curved. For a curved interface, as described before, both the liquid and vapor are in superheated states. The state of a surrounding liquid at temperature T therefore lies along the curve BC; the state of a bubble vapor at temperature T lies on curve EF.

D. Thermodynamic States on a Saturation Curve

Once the equilibrium conditions for a spherical bubble have been described, it is easy to locate the states of liquid and vapor with a curved interface. In Fig. 3, the position L represents the state of liquid, and the position V represents the state of vapor.

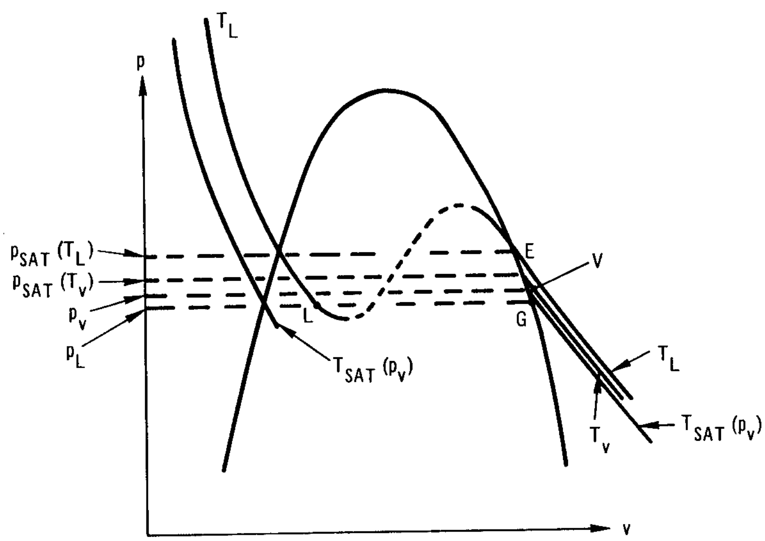

Fig. 3. Equilibrium States of Liquid and Vapor with a Curved Interface

It can be seen that the conditions

$T > T_{sat}(p)$,

$p_V > p$,

and

$T_V > T_{sat}(p_V)$

are satisfied.

For a curved interface, Glasstone[1] and Kast[2] have deduced the following relationship between p_V and $p_{sat}(T_V)$ from thermodynamic reasoning:

$$p_{sat}(T_V) = p_V \exp \frac{2\sigma v}{R p_V v_V} \simeq p_V\left(1 + \frac{2\sigma v}{p_V R v_V}\right). \tag{5}$$

Therefore,

$$p_{sat}(T_V) - p_V \simeq \frac{2\sigma}{R}\frac{v}{v_V}. \tag{6}$$

Substituting Eq. 3 into Eq. 6, we get

$$p_{sat}(T_V) - p \simeq \frac{2\sigma}{R}\left(1 + \frac{v}{v_V}\right). \tag{7}$$

As $v/v_V << 1$,

$$p_{sat}(T_V) \simeq p_V. \tag{8}$$

This implies that the state of vapor V is very close to the saturation curve and that

$$T_V \simeq T_{sat}(p_V). \tag{9}$$

The state of vapor being very close to the saturation curve, we can use the Clausius-Clapeyron equation

$$\frac{dp}{dT} = \frac{L}{T(v_V - v)} \tag{10}$$

to obtain approximate relation for the amount of superheat. If we approximate Eq. 10 and combine it with Eq. 7, we get

$$\frac{p_{sat}(T_V) - p}{T_V - T_{sat}(p)} = \frac{\frac{2\sigma}{R}\left(1 + \frac{v}{v_V}\right)}{T_V - T_{sat}(p)} = \frac{L}{T(v_V - v)}, \tag{11}$$

which, on knowing that $v << v_V$, we can further simplify to

$$T_V - T_{sat}(p) \simeq \frac{2\sigma T v_V}{RL}. \tag{12}$$

Equation 12 gives the minimum amount of superheat required for a bubble of radius R to grow. Any bubble with smaller radius or less superheat will collapse. In Fig. 4, we have compared the superheat requirements for water and for liquid sodium. It can be seen that liquid sodium requires more superheat for the same bubble radii.

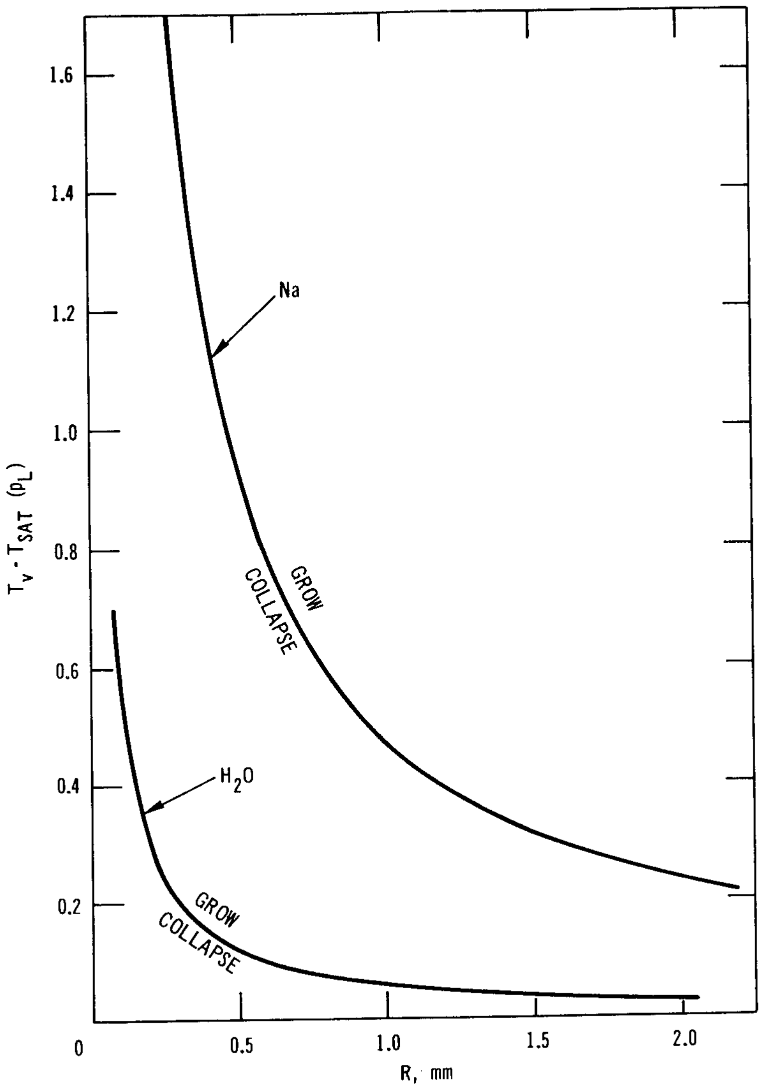

Fig. 4. Growth and Collapse Regions for Sodium and Water-vapor Bubbles

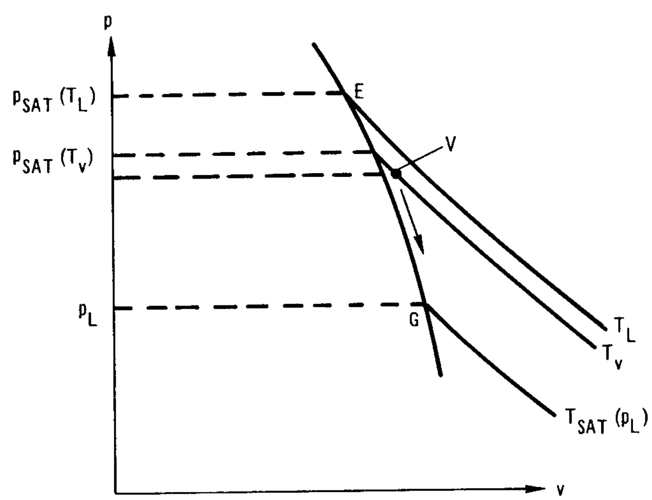

Fig. 5

Equilibrium State for Inertia-controlled Growth

E. Growing Process on Saturation Curve

We are considering the growth of a vapor bubble in a liquid that is at uniform temperature and pressure. It is therefore assumed that the temperature T and pressure p remain constant during the process.

When a bubble grows, its radius R increases, and the pressure p_V and temperature T_V decrease. In addition, we know that the thermodynamic state of vapor is very close to the saturation curve. The state of the vapor therefore moves along the direction as shown in Fig. 5.

The main task of bubble dynamicist is to determine the state of the vapor as a function of time. This requires simultaneous solution of the continuity, momentum, and energy equations for given initial conditions. The problem is very difficult, as many times the initial conditions are not very well known.

Let us look at the two extreme cases.

1. Inertia-controlled Growth

When the state of vapor is very close to E, as shown in Fig. 5, we have

$$p_V \simeq p_{sat}(T), \tag{13}$$

and

$$T_v \simeq T. \tag{14}$$

This means that we have a larger pressure difference $(p_V - p)$ and negligible temperature difference $(T - T_V)$ acting on a spherical interface. Therefore, heat-transfer rates to vapor can be neglected. The growth of a bubble is mainly controlled by inertia force. The solution of the momentum equation alone may be sufficient to determine the growth rate.

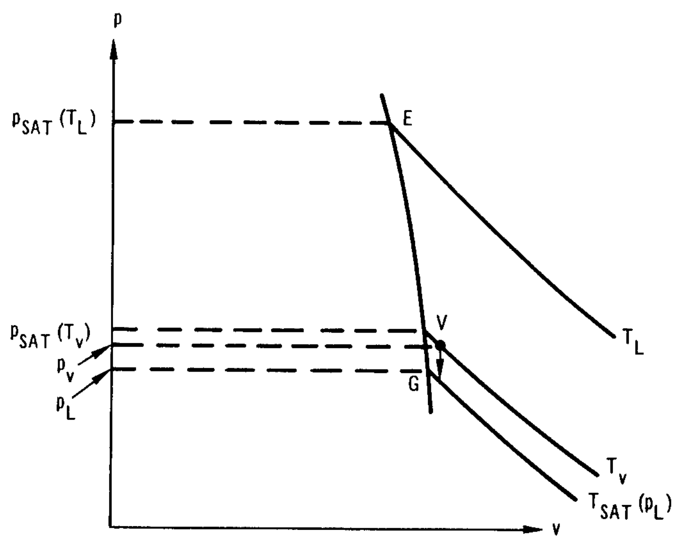

Fig. 6. Equilibrium State for Heat-transfer-controlled Growth

2. Heat-transfer-controlled Growth

When the state of vapor is very close to G, as shown in Fig. 6, we have

$$p_V \simeq p \tag{15}$$

and

$$T_v \simeq T_{sat}(p). \tag{16}$$

In this case, we have a smaller pressure difference $(p_V - p)$ and larger temperature difference $(T - T_V)$. The heat-transfer rate is very high and cannot be neglected. However, the inertia forces, being small, can be neglected. The solution of the energy equation alone may be sufficient to determine the growth rate. The process in this domain is heat-transfer controlled.

III. DYNAMICS OF A STATIONARY BUBBLE

In the last 25 years, considerable work has been done to determine the growth rate (or collapse rate) of a stationary bubble. Most of the theoretical analyses are based on assuming a spherical bubble and uniform vapor properties. Even with these assumptions, the problem remains formidable. It now requires a solution of one-dimensional conservation equations for a surrounding liquid with suitable initial and interface boundary conditions. In this section, we briefly review the results of some of the theoretical analyses after describing the appropriate governing equations.

A. Governing Equations

In presenting the governing equations, we have assumed a spherical symmetry and considered the bulk-viscosity coefficient to be zero.

1. Conservation Equations for a Surrounding Liquid Phase

a. Continuity

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \rho u_r) = 0. \tag{17}$$

For an incompressible fluid, Eq. 17 simplifies to

$$r^2 u_r = \text{constant} = R^2 u_R, \tag{18}$$

or

$$u_r = R^2 u_R / r^2. \tag{18a}$$

b. Momentum Equation

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{\rho}\left(\frac{4\mu}{3} + \beta\right)\frac{\partial}{\partial r}\left(\frac{\partial u_r}{\partial r} + \frac{2u_r}{r}\right) + b. \tag{19}$$

Here, b is the body forces, and β is the second coefficient of viscosity. If the surrounding fluid is incompressible,

$$\frac{\partial u_r}{\partial r} + \frac{2u_r}{r} = 0 \tag{20}$$

from Eqs. 18, and Eq. 19 simplifies to

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + b. \tag{21}$$

If body forces are not present, Eq. 21 can be integrated with the aid of Eq. 18a to obtain

$$2u_R\dot{R} + R\dot{u}_R - \tfrac{1}{2}u_R^2 = \frac{1}{\rho}(p_R - p_\infty), \tag{22}$$

where $\cdot$ represents differentiation with respect to time.

If we further assume that the rate of evaporation is negligible, then the conservation of mass at the interface leads to the relation $u_R = \dot{R}$ (Eq. 30 below). The momentum equation (Eq. 22) now simplifies to

$$R\ddot{R} + \tfrac{3}{2}\dot{R}^2 = \frac{1}{\rho}(p_R - p_\infty), \tag{23}$$

which is Rayleigh's equation. With the aid of momentum balance at the interface (see Eq. 33 below), the motion of bubble surface can be expressed in terms of vapor pressure p_V. Thus,

$$R\ddot{R} + \tfrac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left(p_V - p_\infty - \frac{2\sigma}{R}\right). \tag{24}$$

c. Energy Equation

$$\rho\left(\frac{\partial e}{\partial t} + u_r\frac{\partial e}{\partial r}\right) = -p\left(\frac{\partial u_r}{\partial r} + \frac{2u_r}{r}\right) + \frac{4\mu}{3}\left(\frac{\partial u_r}{\partial r} - \frac{u_r}{r}\right)^2 + k\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right) + \rho\dot{q}''', \tag{25}$$

where $\dot{q}'''$ is the heat source and e is the specific internal energy. If we assume no heat source, no viscous dissipation, negligible kinetic energy, and an incompressible fluid with constant specific heat, Eq. 25 simplifies to

$$\frac{\partial T}{\partial t} + u_r\frac{\partial T}{\partial r} = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right), \tag{26}$$

where $\alpha = k/\rho c$ is the thermal diffusivity.

2. Interface Relations and Boundary Conditions at $r = R(t)$

a. Conservation of Mass. If we consider the total mass balance of a bubble, we get

$$\frac{d}{dt}\left(\tfrac{4}{3}\pi R^3\rho_V\right) = 4\pi R^2\rho(\dot{R} - u_R), \tag{27}$$

which can be rewritten

$$u_R = \epsilon\dot{R} - \dot{\epsilon}R/3, \tag{28}$$

where

$$\epsilon = 1 - \frac{\rho_v}{\rho}. \tag{29}$$

If it is assumed that $\rho_v << \rho$, then $\epsilon \simeq 1$ and $\dot{\epsilon} \simeq 0$, and Eq. 28 simplifies to

$$u_R = \dot{R}. \tag{30}$$

Equation 30 implicitly assumes that the rate of evaporation is negligible.

b. Momentum Balance. For a concave interface, the vapor pressure p_v is related to the liquid pressure by

$$p_v = p_R + \frac{2\sigma}{R} - \tau_{rr} + \rho_v u_{R_v}(u_{R_v} - \dot{R}) - \rho u_R(u_R - \dot{R}). \tag{31}$$

If we ignore momentum fluxes at the interface,

$$p_v = p_R + \frac{2\sigma}{R} - \frac{4}{3}\mu\left(\frac{\partial u_r}{\partial r} - \frac{u_r}{r}\right)_R. \tag{32}$$

With the further assumption of negligible viscous stress, Eq. 32 simplifies to

$$p_v = p_R + \frac{2\sigma}{R}. \tag{33}$$

c. Energy Balance. The energy balance at the interface leads to

$$k\frac{\partial T}{\partial r} = \frac{1}{4\pi R^2}\frac{d}{dt}\left(\frac{4}{3}\pi R^3 \rho_v\right)\left[L + \frac{4\mu}{3}\left(\frac{\partial u_r}{\partial r} - \frac{u_r}{r}\right)\right] + \frac{\rho_v R}{3}\frac{du_{R_v}}{dt} - \frac{p_v R}{3\rho_v}\frac{d\rho_v}{dt}. \tag{34}$$

In Eq. 34, it is assumed that the kinetic terms are negligible. In most situations, the latent heat term usually dominates and Eq. 34 simplifies to

$$k\frac{\partial T}{\partial r} = \frac{L}{4\pi R^2}\frac{d}{dt}\left(\frac{4}{3}\pi R^3 \rho_v\right). \tag{35}$$

d. Surface Temperature. The boundary condition at the interface will be

$$T_R = T_v, \tag{36}$$

as it is assumed that the properties of vapor inside the bubble are uniform.

B. Brief Review of Past Work

It can be seen from the previous section that the bubble growth rate can be determined from the solution of Eq. 24. The vapor pressure $p_V(t)$ in Eq. 24 depends on vapor temperature T_V (= T_R), which, in turn, depends on the heat-transfer rate at the bubble surface. It is, therefore, necessary to solve Eq. 26 along with Eq. 24.

The bubble growth rate is essentially controlled by inertial, thermal, and diffusive effects. Here we are considering only the first two effects. Initially, the rate of growth of a vapor bubble is controlled by the liquid inertia, surface tension, and the pressure difference ($p_V - p_\infty$). At the start of the expansion of the original bubble nucleus, the forces are nearly in equilibrium and the bubble growth rate is slow. As the bubble starts growing, the surface tension force reduces and the growth rate is accelerated. Within a very short time, the growth becomes appreciable and the temperature and pressure of vapor drop. The growth rate now decreases and is essentially controlled by the thermal effects. The bubble will either continue growing or start collapsing, depending on whether the surrounding fluid is superheated or subcooled. The qualitative description of this process is shown in Figs. 7 and 8.

When the inertia forces are dominating, the changes in vapor pressure can be neglected and the solution of Eq. 24 is sufficient to predict the bubble growth rate. The results are valid in the initial stages of growth and for low

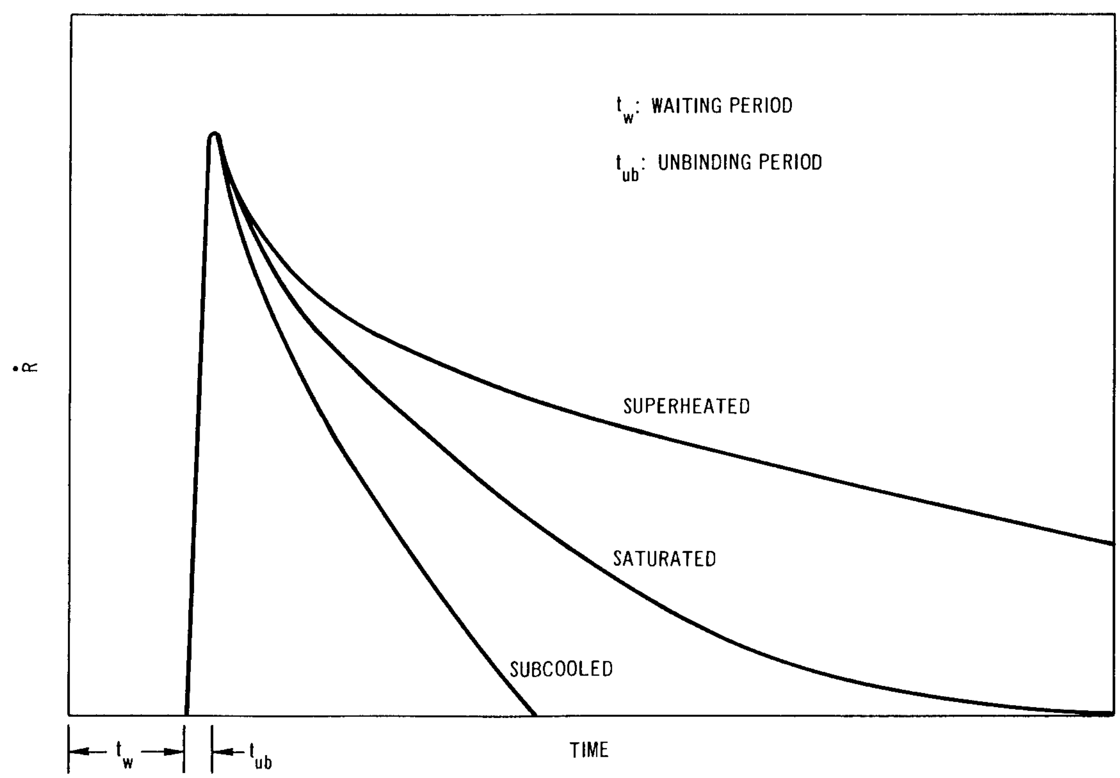

Fig. 7. Variation of Growth Rate in Superheated, Saturated, and Subcooled Regions

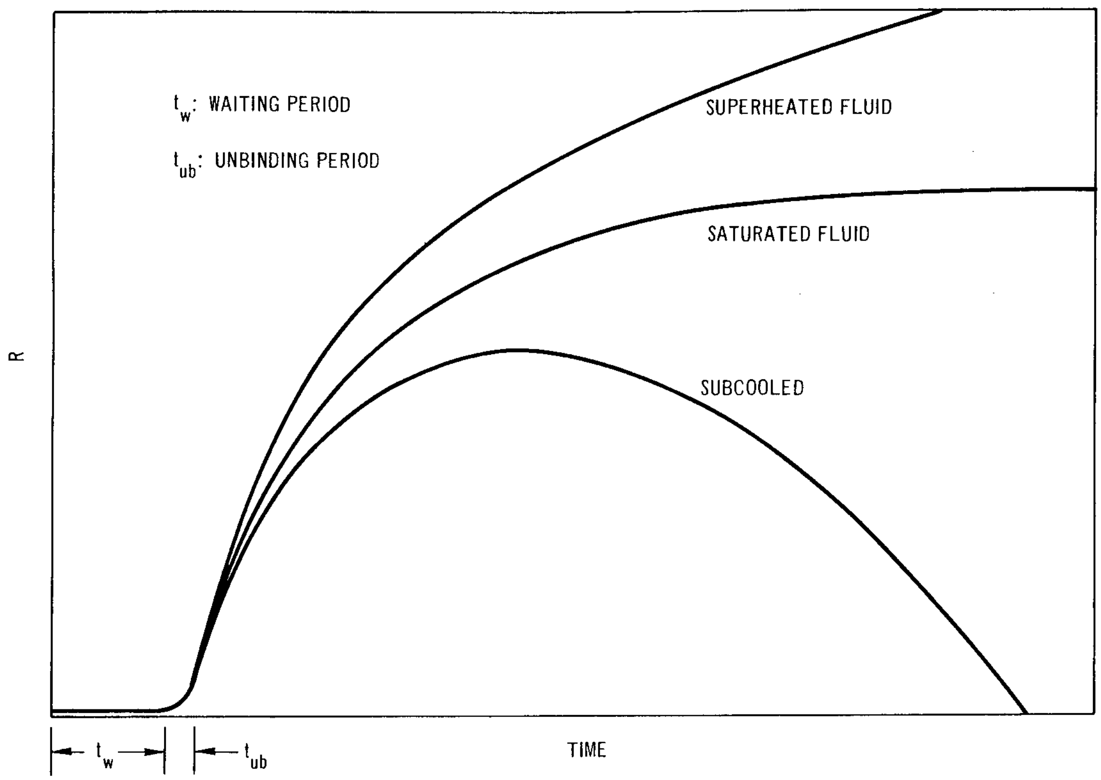

Fig. 8. Variation of Bubble Radius in Superheated, Saturated, and Subcooled Regions

Jakob numbers. On the other hand, when the thermal effect is dominant, the solution of Eq. 26 and the interface relation (Eq. 35) are sufficient to determine the growth rate. Since the simultaneous solution of Eqs. 24 and 26 has been difficult, most of the analyses are restricted either to inertia-controlled or to heat-transfer-controlled growth.

1. Inertia-controlled Growth

Rayleigh[3] was one of the first researchers in bubble dynamics. He assumed a uniform pressure difference and integrated Eq. 23 to predict that the radius of a bubble increases linearly with time; i.e.,

$$R \propto t \text{ (Rayleigh's solution).} \tag{37}$$

This solution is applicable when the bubble-wall motion is slow and the liquid can be treated as incompressible. Dergarabedian[4] analyzed Rayleigh's equation accounting for the effect of surface tension, but assumed a constant value for vapor pressure. Hunter,[5] Hickling and Plesset,[6] and Hsieh[7] extended the solution of the momentum equation to account for compressibility effects.

2. Heat-diffusion-controlled Growth

a. Uniform Temperature Field. Plesset and Zwick[8] derived an integro-differential equation by combining the momentum equation (Eq. 24) and the solution of the energy equation (Eq. 26) with an assumption of a thin

thermal-boundary layer near the bubble wall. They obtained an approximate solution by dividing the growth period into several regimes and using possible simplification for each regime. Their solution

$$R = cJa\sqrt{\alpha t} \tag{38}$$

$$\left(c = 2\sqrt{\frac{3}{\pi}} = 1.95\right)$$

is an asymptotic solution when a bubble becomes large. Equation 38 is found to be in agreement with the experimental measurements even near the lower limit.

Forster and Zuber[9] also obtained an approximate solution of the integro-differential equation:

$$R + R_0 \log_e \frac{\frac{R}{R_0} - 1}{\frac{R_1}{R_0} - 1} = cJa\sqrt{\alpha t} \tag{39}$$

$$\left(c = \sqrt{\pi} = 1.77\right).$$

Here, R_0 is the initial radius in equilibrium with the surrounding fluid, and R_1 is the radius at which the rates of growth of the bubble resulting from the superheat term and from the evaporation term attain an equal order of magnitude. Since R_0 is very small, Eq. 39 is in agreement with the Plesset and Zwick solution given by Eq. 38.

Birkhoff et al.[10] considered a similar solution of Eq. 26. They also obtained a solution of the form given by Eq. 38, with the coefficient c being an implicit function of the Jakob number:

$$R = c(Ja)Ja\sqrt{\alpha t}. \tag{40}$$

For a large Jakob number, the coefficient c asymptotically approaches the value of Plesset and Zwick.[8]

Scriven[11] extended the solution of Eq. 26 without neglecting the effect of density ratio ρ_v/ρ. His solution is of the same form as that of Plesset and Zwick, except that the growth coefficient c is an implicit function of the Jakob number. He presented graphs and tables of growth coefficient for a range of Jakob numbers and for a range of density ratios.

b. Nonuniform Temperature Field. Bankoff and Mikesell[12] used the Plesset and Zwick perturbation technique to obtain the growth of a spherical bubble in initial temperature fields that were exponential and ramp functions of the Lagrangian radial coordinates.

Griffith[13] considered the growth of a hemispherical bubble at the heated surface. He obtained the finite-difference solution of the energy equation and applied the heat flux at the bubble surface to determine the growth rate. He obtained the numerical solution for three Jakob numbers (5.3, 1, and 0.35) and for three values (0, 0.67, and 0.9) of dimensionless temperature:

$$\theta = (T_{sat} - T)/(T_w - T).$$

Savic[14] used a model similar to that of Griffith, but ignored the variation of temperature with the polar angle. He applied the perturbation technique to solve the energy equation and obtained the first three terms of the power series

$$R = 1.95 Ja \sqrt{\alpha t} + B\frac{R}{\dot{R}} + D'\frac{R^2}{\dot{R}}, \tag{41}$$

where the values of B and D' depend on the fluid properties. Savic has obtained the relation for computing the values of these constants.

Zuber[15] assumed that the heat-flux contribution to bubble growth is less in a nonuniform temperature field than in a uniform temperature field. He modified Eq. 35 by the amount of an average heat flux $\dot{q}_b$ from the boiling surface

$$L\rho_v \frac{dR}{dt} = \left(k\frac{\partial T}{\partial r}\bigg|_R\right)_{\text{uniform field}} - \dot{q}_b. \tag{42}$$

He integrated Eq. 42, using the relation of Fritz and Ende,[16]

$$\left(k\frac{\partial T}{\partial r}\bigg|_R\right)_{\text{uniform field}} = k\frac{\Delta T}{\sqrt{\pi\alpha t}} \tag{43}$$

and obtained the growth rate

$$R = \frac{2}{\sqrt{\pi}} Ja \sqrt{\alpha t}\left(1 - \frac{\dot{q}_b\sqrt{\pi\alpha t}}{2k\Delta T}\right). \tag{44}$$

Skinner and Bankoff[17] analyzed Eq. 26 for a general spherically symmetric initial condition. The general solution was applied to a specific case of nucleate boiling on a heated surface (the initial temperature distribution of the exponential form), and the growth rates plotted for a range of subcooled and superheated temperature ratios.

Han and Griffith[18] considered the transient conduction equation for a layer of liquid on the surface. They used the solution of this equation and the heat flux at the surface to determine the growth rate of a bubble. Their expression for bubble radius,

$$R = R(t), \tag{45}$$

is somewhat involved for a simple numerical computation.

Cole and Shulman[19] compared experimental data with some existing theories for Jakob numbers ranging from 24 to 792. They observed that the growth data over this range can be correlated by

$$R = 2.5(Ja)^{3/4}\sqrt{\alpha t}. \tag{46}$$

Van Stralen[20] modified the Plesset and Zwick solution by assuming that the wall superheat is a function of time and that only part of the bubble is covered by superheated layer. His expression for the growth rate is

$$R = 2bJa\left(\frac{3\alpha t}{\pi}\right)^{1/2} \exp[-(t/t_1)^{1/2}], \tag{47}$$

where b is a correction factor of value less than one, and t_1 is the time at which the bubble is breaking away from a heating surface.

Mikic and Rohsenow[21] expressed the nonuniformity as a function of waiting time and applied the solution of one-dimensional transient conduction equation to obtain an expression for the growth rate:

$$R = 2Ja\left(\frac{3\alpha t}{\pi}\right)^{1/2}\left\{1 - \frac{T_W - T}{T_W - T_{sat}}\left[\left(1 + \frac{t_W}{t}\right)^{1/2} - \left(\frac{t_W}{t}\right)^{1/2}\right]\right\}, \tag{48}$$

where t_W is the waiting period, which is a function of radius of cavity.

3. Inertia- and Heat-diffusion-controlled Growth

Mikic et al.[22] extended the results to include the effect of inertia during the initial period of growth and obtained the following expression for the growth rate:

$$\frac{dR^+}{dt^+} = \left[t^+ + 1 + \theta\left(\frac{t^+}{t^+ + t_W^+}\right)^{1/2}\right]^{1/2} - (t^+)^{1/2}, \tag{49}$$

where

$$R^+ = \frac{\pi \Delta T}{12\alpha}\left(\frac{bc}{T_{sat}}\right)^{1/2}\frac{1}{(Ja)^{5/2}}R, \tag{50}$$

$$t^+ = \frac{\pi}{12}\,\frac{bc}{\alpha T_{sat}}\,\frac{(\Delta T)^2}{(Ja)^3}t, \tag{51}$$

and

$$\theta = \frac{T_W - T}{T_W - T_{sat}}. \tag{52}$$

The constant b is equal to 2/3 for a bubble growing in an infinite mass of liquid, and is $\pi/7$ for a spherical bubble growing attached to a surface.

Equation 49 can be integrated for the parameters θ and t_W^+. For growth in a uniformly superheated liquid ($t_W \rightarrow \infty$), it gives

$$R^+ = \tfrac{2}{3}[(t^+ + 1)^{3/2} - (t^+)^{3/2} - 1]. \tag{53}$$

Theofanous et al.[23] extended the work of Ref. 22 by accounting for the variation of vapor density. They assumed a linear pressure-temperature relation for the saturation curve and obtained results for initial to final vapor-density ratios up to 20.

Dalle Donne and Ferranti[24] numerically integrated both the energy and the momentum differential equations, to determine the growth of a sodium-vapor bubble. They took into account the variations of sodium properties with temperature and obtained results for a range of pressures, superheats, and Jakob numbers. Their results indicate that, even during the initial transient period, the heat transfer has the significant influence on the growth rate, the effects of all other factors being less than 25%.

Prosperetti[25] reexamined the Plesset and Zwick solution to compare the results of Dalle Donne and Ferranti. He observed that, as soon as the initial effects become negligible (i.e., when the radius of a bubble is approximately 10 times the initial radius), the bubble growth can be described by a universal function applicable to any liquid and any superheat.

C. Growth of a Sodium-vapor Bubble

To determine the order of magnitude of a sodium-vapor bubble, Eq. 53 has been used to compute the growth rate of a vapor bubble in a uniform temperature field. The physical-property values of sodium were obtained from the tables and graphs of Golden and Tokar.[26] The effects of pressure and superheat on the growth rate are shown in Figs. 9 and 10. Figure 11 compares the growth rates of water and sodium vapor bubbles. It can be seen that for the same values of superheat, a sodium-vapor bubble grows faster than a water-vapor bubble.

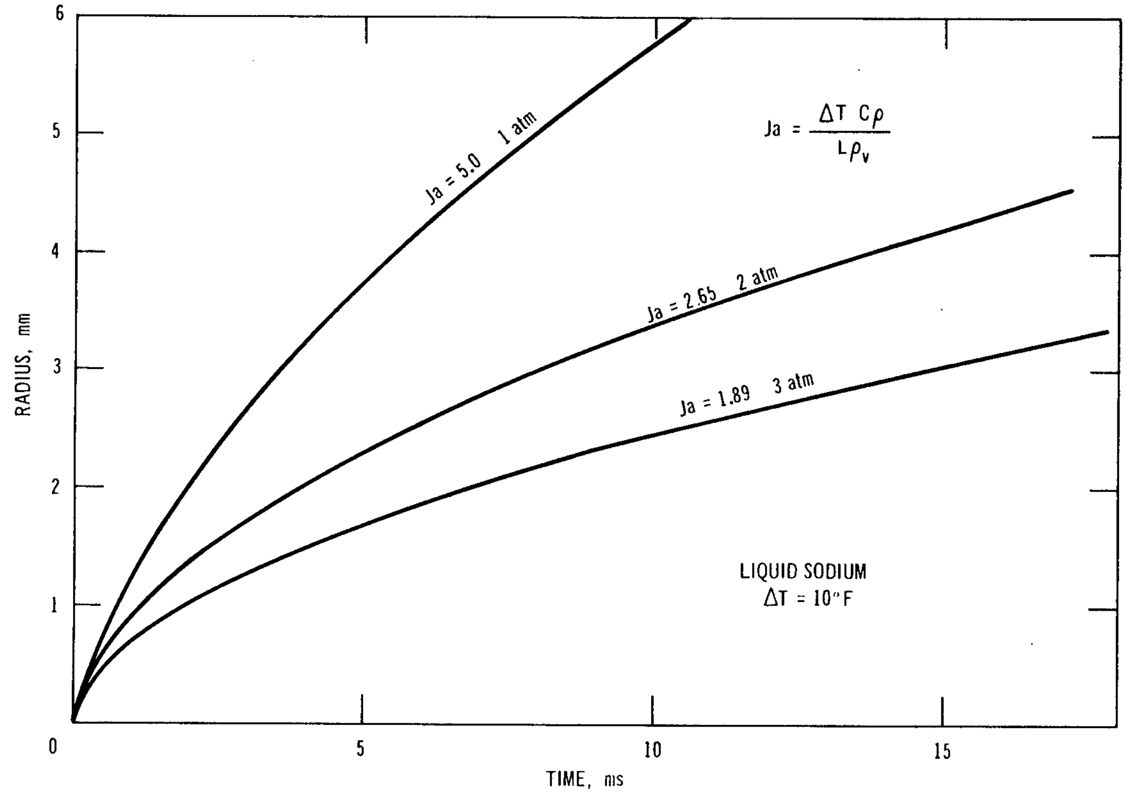

Fig. 9. Effect of Pressure on Growth of a Sodium-vapor Bubble. Conversion factors: 1 atm = 0.1 MPa; °C = (°F − 32)/1.8.

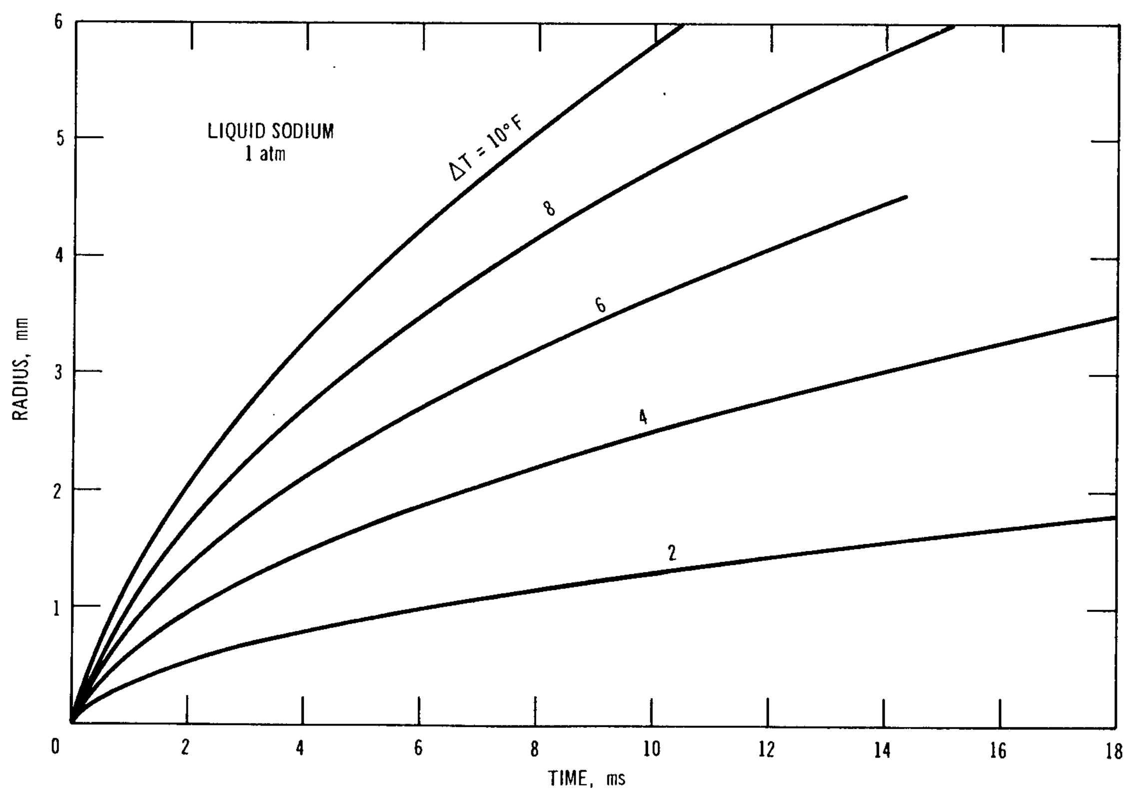

Fig. 10. Effect of Superheat on Growth of a Sodium-vapor Bubble. Conversion factors: 1 atm = 0.1 MPa; °C = (°F − 32)/1.8.

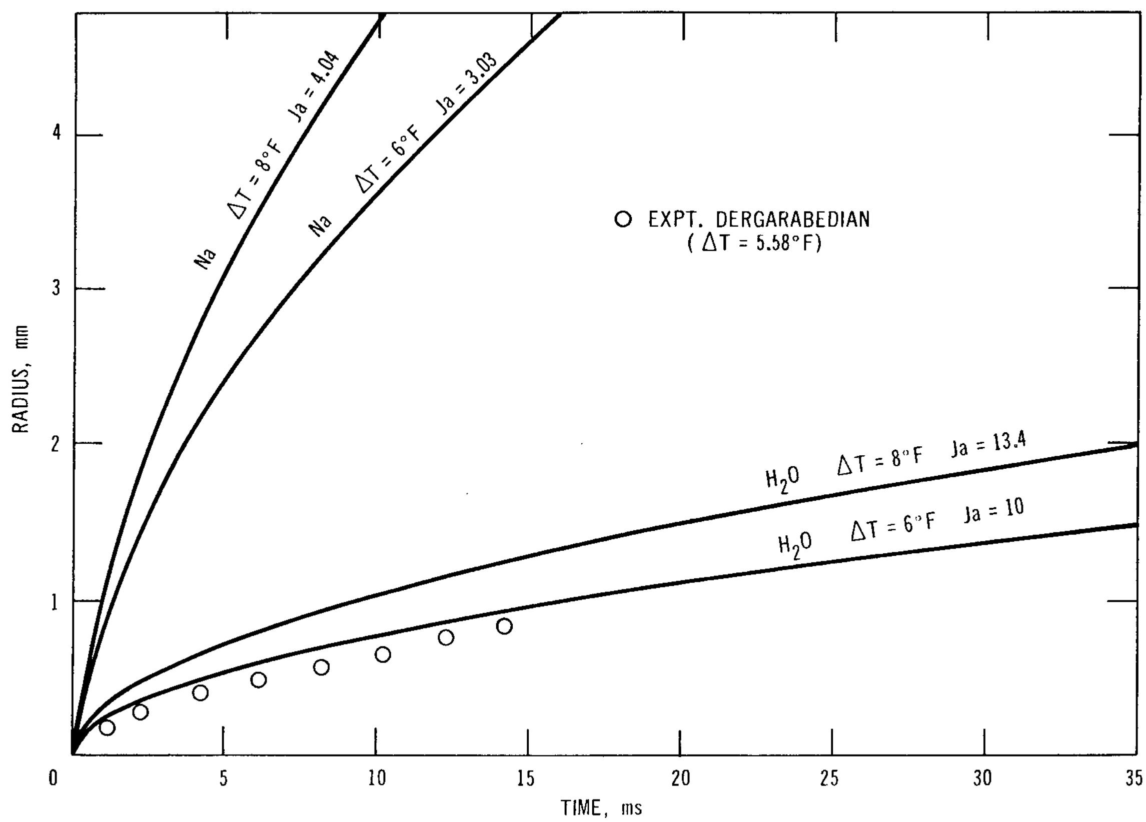

Fig. 11. Comparison of Growth Rates of Water- and Sodium-vapor Bubbles. Conversion factor: °C = (°F − 32)/1.8.

IV. MOTION OF A BUBBLE IN A VISCOUS LIQUID

Considerable theoretical and experimental effort has been focused on the dynamics of a moving bubble. It has been inferred from past investigations that:

1. For small bubbles at low Reynolds numbers, the shape of a bubble is nearly spherical.

2. For extremely low Reynolds numbers (Re << 1), bubbles rise like solid spherical particles. Stoke's solution for a flow past a solid sphere is applicable in this regime.

3. At moderate Reynolds numbers, the motion of a bubble can be considered as similar to, but less complicated than, the motion of a liquid droplet in a liquid medium. We can, therefore, apply the solutions for fluid spheres[27-29] to determine the motion of a spherical gas bubble in the low-Reynolds-number regime.

4. At high Reynolds numbers ($Re \simeq 1000$), bubbles deform from a spherical to an oblate shape. In this regime, bubbles vibrate and rise along a spiral path.

5. With a further increase in Reynolds number, bubbles attain an umbrella shape (spherical cap) that is quite steady.

The dimensionless parameters generally used to describe the characteristics of a moving bubble are

$$M = g\mu^4/\rho\sigma^3, \tag{54}$$

$$Re = 2\rho u_\infty R/\mu \text{ (Reynolds number)}, \tag{55}$$

$$W = 2\rho u_\infty^2 R/\sigma \text{ (Weber number)}, \tag{56}$$

$$F = u_\infty^2/2gR \text{ (Froude number)}, \tag{57}$$

and

$$E = 4gR^2\rho/\alpha \text{ (Eötvös number)}, \tag{58}$$

where u_∞ is the steady rising velocity, also known as terminal velocity, of a bubble.

A. Governing Equations

The problem of a spherical bubble rising with constant velocity u_∞ is the same as that of a stationary sphere situated in a downward flowing liquid with a constant velocity u_∞ at infinity. The continuity and the Navier-Stokes equation of motion for a liquid with constant density ρ and viscosity μ are:

Continuity:

$$\nabla \cdot \bar{u} = 0. \tag{59}$$

Momentum:

$$(\bar{u} \cdot \nabla)\bar{u} = \bar{g} - \frac{1}{\rho}\nabla p + \nu\nabla^2\bar{u}. \tag{60}$$

The corresponding boundary conditions are:

$$\bar{u} \rightarrow u_\infty \text{ as } |r| \rightarrow \infty, \tag{61}$$

$$\bar{u} \cdot \bar{n} = 0 \text{ on } \bar{r} = R, \tag{62}$$

and

$$\tau_{r\theta} = 0 \quad \text{on} \quad \bar{r} = R, \tag{63}$$

where $\bar{n}$ is a unit normal vector.

B. Brief Review of Some Theoretical Work

Stokes[30] considered the problem of the very slow motion (creeping motion; Re << 1) of a sphere. He neglected the inertial effects and obtained the solution of the equation of motion (Eq. 60), satisfying Eqs. 61 and 62, and the no-slip boundary condition $u_\theta = 0$ at $r = R$. The final results for terminal velocity and viscous drag were

$$\tau_{r\theta}\Big|_R = \frac{3}{2}\,\frac{\mu u_\infty}{R}\sin\theta, \tag{64}$$

$$u_\infty = \frac{2}{9}\,\frac{R^2 g(\rho' - \rho)}{\mu}, \tag{65}$$

and

$$C_D = \frac{24}{Re}, \tag{66}$$

where ρ' is the density of a sphere.

Hadamard[27] and Rybczynski[28] considered the motion of a liquid droplet. They also ignored the inertia terms from the equation of motion, but their solutions satisfy all three boundary conditions expressed by Eqs. 61-63. Their final results were

$$u_\infty = \frac{2(\rho' - \rho)gR^2}{3\mu}\,\frac{\mu + \mu'}{2\mu + 3\mu'}, \tag{67}$$

$$(u_\theta)_R = \frac{\mu}{2}\,\frac{u_\infty}{(\mu + \mu')}\sin\theta, \tag{68}$$

and

$$C_D = \frac{24}{Re}\,\frac{2\mu + 3\mu'}{3\mu + 3\mu'}, \tag{69}$$

where μ' is the viscosity of a droplet fluid. When μ' is very large ($\mu' >> \mu$; rigid sphere), the results of Hadamard and Rybczynski are in agreement with the Stokes solution. These results are also applicable to gas bubbles ($\mu' \sim 0$) in the low-Reynolds-number regime.

Boussinesq[29] also considered the motion of a droplet. He assumed that a thin layer near the interface has a higher viscosity. He then solved the equations of motion to obtain

$$u_\infty = \frac{2}{9}\,\frac{\rho - \rho'}{\mu}\, gR^2 \frac{3\mu + 3\mu' + 2e/R}{2\mu + 3\mu' + 2e/R}, \tag{70}$$

and

$$C_D = \frac{24}{Re}\,\frac{2\mu + 3\mu' + 2e/R}{3\mu + 3\mu' + 2e/R}. \tag{71}$$

For large drop radii, Eq. 71 is in agreement with the results of Hadamard and Rybczynski.

Levich[31] analyzed the motion of a gas bubble for $Re \gg 1$. He assumed that there is a thin boundary layer near the interface and that the velocity field in this layer is slightly different from the velocity field for inviscid flow past a sphere. With these assumptions, he obtained the solution of the equation of motion that satisfied all three boundary conditions. His solution for the perturbed velocity field is in the form of an integral. The drag coefficient and the corresponding terminal velocity are computed by integrating the dissipation energy:

$$C_D = 48/Re \tag{72}$$

and

$$u_\infty = \rho g R^2/8\mu. \tag{73}$$

It can be seen that the drag coefficient is twice and the terminal velocity is half the corresponding values of Stokes' flow.

Moore[32] assumed that at high Reynolds numbers the flow field surrounding a spherical bubble is irrotational; i.e.,

$$u_r = u_\infty(1 - R^3/r^3)\cos\theta \tag{74}$$

and

$$u_\theta = u_\infty(1 + R^3/2r^3)\sin\theta. \tag{75}$$

Although this velocity field does not satisfy the zero-shear-stress boundary condition of Eq. 63, Moore used this solution to compute the drag coefficient on the ground that the drag force arises entirely from the normal viscous stresses. He obtained

$$u_\infty = \rho g R^2/6\mu, \tag{76}$$

and

$$C_D = 32/\mathrm{Re}, \tag{77}$$

which appears to be in fair agreement with experimental data at moderate Reynolds numbers.

Similar to Levich, Chao[33] assumed a thin boundary layer in the vicinity of interface and the velocity field slightly perturbed from the potential (irrotational) flow solution. He included the effects of internal circulation and also corrected the continuity relationship used by Levich. His solutions for the perturbed velocity are in the form of the first integral of the complimentary error function. He also interpreted the stress field at the interface to obtain the drag coefficient

$$C_D = \frac{32}{\mathrm{Re}}\left(1 + \frac{2\mu'}{\mu} - 0.314\,\frac{1 + 4\mu'/\mu}{\mathrm{Re}^{1/2}}\right), \tag{78}$$

which, for $\mu' \sim 0$, reduces to

$$C_D = \frac{32}{\mathrm{Re}}\left(1 - \frac{0.314}{\sqrt{\mathrm{Re}}}\right). \tag{79}$$

Moore[34] reexamined the motion of a spherical gas bubble to include the effects of pressure forces in the boundary layer for computation of the drag coefficient. For the surface component of the velocity, Moore arrived at the equation that is identical to Chao's boundary-layer equation. Their solutions are slightly different, as Chao had an error in one of his boundary conditions. Moore obtained an expression for the drag coefficient, using the momentum argument, in which he included the contribution from the boundary layer and wake to the dissipation calculation. His expression for the drag coefficient is

$$C_D = \frac{48}{\mathrm{Re}}\left[1 - \frac{2.211}{(\mathrm{Re})^{1/2}} + O(\mathrm{Re}^{-5/6})\right]. \tag{80}$$

In this section, we have restricted our review of some theoretical work about the motion of a spherical bubble. At very high Reynolds numbers, the shape of a bubble no longer remains spherical. Although we have not reviewed it here, considerable work has been done to evaluate the motion of a distorted bubble. Harper[35] has published a comprehensive article that includes some theoretical results for spheroidal and spherical cap bubbles.

Since our main interest is to study the behavior of a sodium-vapor bubble, we have presented in Fig. 12 a graph of terminal velocity as a function of radius for a sodium-vapor bubble, and in Fig. 13 the drag coefficient as a function of Reynolds number.

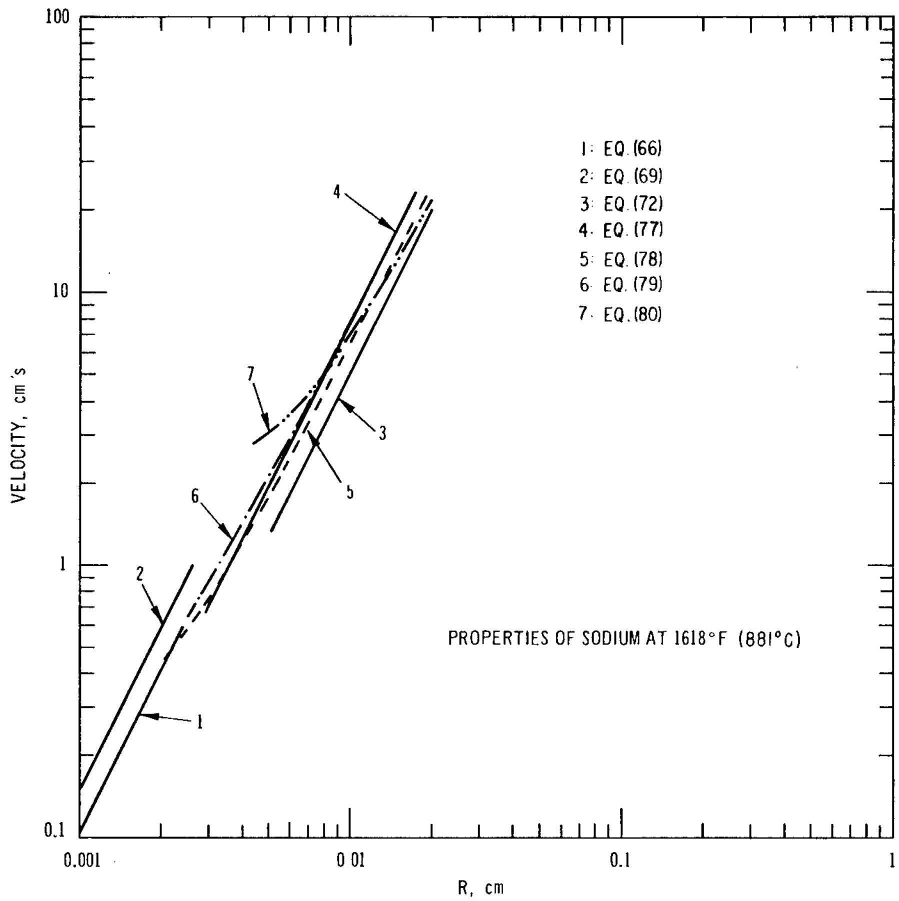

Fig. 12. Terminal Velocity of a Sodium-vapor Bubble

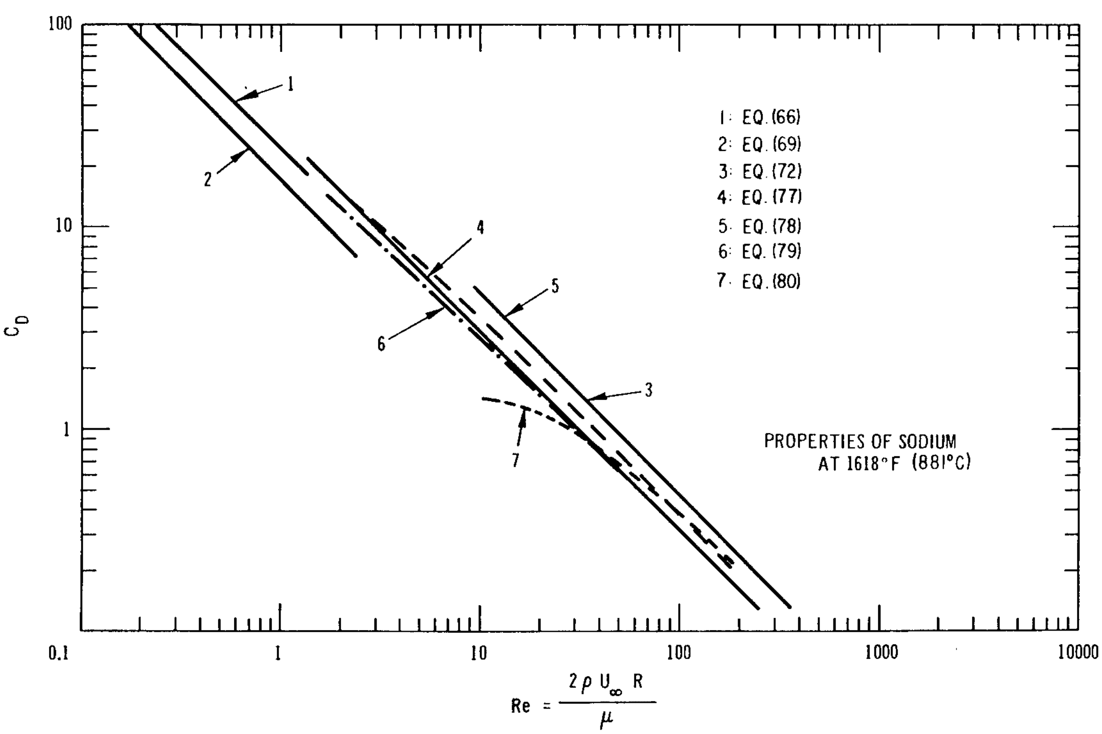

Fig. 13. Drag Coefficient for a Sodium-vapor Bubble

V. DYNAMICS OF A MOVING BUBBLE

We have seen from the previous two sections that considerable work has been done for the cases of (1) a bubble growing but stationary, and (2) a moving bubble of constant volume. Recently, some attempts have been made to analyze the growth of a bubble that is simultaneously in motion.

The thermal-boundary-layer thickness of a bubble in motion (rising) is smaller than that for a stationary bubble. As the bubble velocity increases, the boundary-layer thickness decreases, and the heat-transfer and growth rates increase. As the radius of a bubble increases, the drag force increases, and the velocity decreases. This change in velocity, in turn, affects the growth rate. Therefore, we can see that for a moving bubble, the equations of motion and energy are coupled and require simultaneous solution.

A. Governing Equations

Consider a spherical coordinate system fixed at the center of a bubble that is moving (rising) with a velocity u_∞ in an incompressible liquid medium of constant viscosity. Assume that the flow is symmetric with respect to the vertical axis passing through the center of the bubble. The governing equations are:

Continuity:

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial\theta}(u_\theta \sin\theta) = 0. \tag{81}$$

Momentum:

$$\frac{\partial\bar{u}}{\partial t} + (\bar{u}\cdot\nabla)\bar{u} = \bar{g} - \frac{1}{\rho}\nabla p + \nu\nabla^2\bar{u}. \tag{82}$$

Energy:

$$\frac{\partial T}{\partial t} + u_r\frac{\partial T}{\partial r} + \frac{u_\theta}{r}\frac{\partial T}{\partial\theta} = \alpha\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)\right]. \tag{83}$$

The corresponding boundary conditions are:

$$\bar{u} \to u_\infty;\quad T \to T_\infty \text{ as } |r| \to \infty;$$

$$u_r = u_R \text{ (Eq. 85)};\quad \tau_{r\theta} = 0;$$

and

$$T = T_{sat} \text{ at } r = R. \tag{84}$$

The integration of mass, momentum, and energy equations at the bubble interface provides

$$u_R = \epsilon\dot{R} - \dot{\epsilon}R/3, \tag{85}$$

$$D = 2\pi R^2 \int_0^{\pi} (\tau_{rr})_R \sin\theta \, d\theta, \tag{86}$$

and

$$\dot{R} = \frac{k}{2\rho_v L} \int_0^{\pi} \left(\frac{\partial T}{\partial r}\right)_R \sin\theta \, d\theta, \tag{87}$$

where τ_{rr} is the normal stress, D is the drag force, and

$$\epsilon = \left(1 - \frac{\rho_v}{\rho}\right). \tag{88}$$

If we ignore the change in the inertia of a bubble, the drag force can be equated to the buoyancy force; then,

$$D = \tfrac{4}{3}\pi R^3 g(\rho - \rho_v) \tag{89}$$

or

$$C_D = \frac{D}{\frac{1}{2}\rho u_\infty^2 \pi R^2} = \frac{8}{3}\frac{Rg\epsilon}{u_\infty^2}. \tag{90}$$

B. Past Work

Ruckenstein[36] used only the energy equation to determine the heat-transfer rate between a translating vapor bubble and the surrounding liquid. He assumed a thin boundary layer near the interface and a steady velocity field corresponding to the inviscid flow solution. Following the method of Levich, he solved the steady-state energy equation to obtain

$$\dot{Q} = 8\Delta T k \left(\frac{\pi}{3}\right)^{1/2} \left(\frac{u_\infty}{2R\alpha}\right)^{1/2} R^2 \tag{91}$$

for the overall heat-transfer rate, and

$$R^{5/4} = R_0^{5/4} + 1.275 Ja \sqrt{\alpha}\, (\epsilon g/C_D)^{1/4} t \tag{92}$$

for the bubble radius. Here, R_0 is the bubble radius at the time of detachment.

Tokuda et al.[37] considered the transient energy and diffusion equations to study the dynamics of a moving bubble. They assumed a potential flow model for the velocity field around the bubble and used the coordinate perturbation technique to develop two asymptotic (small-time and large-time) expressions for the growth rate. They integrated these expressions numerically to obtain the radius as a function of time. Their analysis confirmed that a translative bubble motion causes a significant increase in the growth rates over those predicted for stationary growing bubbles.

Chao[38] made an analysis for the transient response behavior of thermal or concentration boundary layers for a liquid droplet moving at constant velocity. He also assumed a thin boundary layer at the interface and a potential flow model for the velocity field, but included the effects of internal circulation. He used similarity transformations and obtained a solution of the transient-energy (or mass-concentration) equation in the form of a complimentary error function. He observed that the overall heat-transfer rate is a function of the parameter $u_\infty t/R$, and that when this parameter attains a value of unity, the transient solution is close (within 1.5%) to the steady-state (large-time) solution.

Ruckenstein and Davis[39] analyzed the transient-energy equation to determine the effects of bubble translation on its growth rate. They also assumed a thin boundary layer surrounding the bubble surface. They assumed the velocity field to be a sum of the potential flow field and the flow field due to pure radial growth. They solved the energy equation, using similarity transformations. Their expressions for the growth rate and bubble radius are rather involved integral equations. They also presented a quasi-steady-state approximation for the growth rate that can be integrated numerically, provided the translational velocity can be specified as a function of radius. They assumed

$$u_\infty = \left(\frac{\sigma}{R\rho} + gR\right)^{1/2} \tag{93}$$

to compare their quasi-steady-state approximation with the results of Florschuetz et al.[40]

Pinto and Davis[41] used Basset's one-dimensional equation of motion

$$\frac{d}{dt}\left[\rho_V\left(\tfrac{4}{3}\pi R^3\right)u + \tfrac{1}{2}\rho\left(\tfrac{4}{3}\pi R^3\right)u\right] = (\rho - \rho_V)\tfrac{4}{3}\pi R^3 g - C_D\pi R^2\rho\frac{u^2}{2} \tag{94}$$

along with the Plesset and Zwick equation[8] to determine the velocity of a growing bubble. The second term in Eq. 94 represents the momentum of liquid accelerated with the rising bubble. Pinto and Davis neglected the terms containing ρ_V in the equation of motion and used the steady-state terminal-velocity/drag-coefficient correlations

$$C_D = \frac{216\nu^2}{gR^3} \text{ (Levich}^{31}) \quad \text{for} \quad R \le 0.04 \text{ cm.} \tag{95}$$

and

$$C_D = \frac{8}{3}\frac{R^2}{1.82}\frac{\rho g}{\sigma} \text{ (Peebles and Garber}^{42}) \tag{96}$$

to predict the rising velocity of a growing bubble. They obtained an analytical solution for smaller bubbles and a numerical solution for larger bubbles. Their results show that, in the initial stages, the gravitational force is the dominant force which increases the rising velocity of a bubble. But as the bubble grows larger, the drag force increases and, consequently, the velocity starts to decrease.

Chang[43] considered the growth of a spherical second phase (bubble or droplet) as governed by simultaneous heat- and mass-transfer limitations. He showed that by using the proper compatibility conditions, one can apply the solution for an uncoupled case (either heat or mass transfer limited) to determine the growth rate when both heat and mass transfer control the growth of a bubble.

C. Order-of-magnitude Approach

Aleksandrov et al.[44] used the order-of-magnitude approach to determine the rate of growth of bubbles that are rising in a superheated liquid. Since our solution is based on their final formula, we are presenting its brief derivation here.

Let δ be the distance from the surface where the temperature of the liquid changes significantly, so that, to an order of magnitude,

$$\frac{\partial T}{\partial r} \simeq \frac{T_\infty - T_{sat}}{\delta} \simeq \frac{\Delta T}{\delta}. \tag{97}$$

The changes of liquid temperature along the bubble surface occur over a distance that is of the order of a bubble radius R; therefore,

$$\frac{1}{r}\frac{\partial T}{\partial \theta} \simeq \frac{\Delta T}{R}. \tag{98}$$

From the hydrodynamic solution of a rising bubble, the orders of magnitude of liquid velocity components near the bubble surface are

$$\left.\begin{aligned} &u_r \simeq 0 \\ \text{and} \\ &u_\theta = \frac{u_\infty \sin\theta}{2} \simeq u_\infty \end{aligned}\right\}. \tag{99}$$

With these assumptions, we can estimate the order of magnitude of each term of the energy equation

$$\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r}\frac{\partial T}{\partial \theta} = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{1}{r^2}\cot\theta\,\frac{\partial T}{\partial \theta}\right). \tag{100}$$

$$\left[\frac{\Delta T}{t} + O\left(\frac{\Delta T}{\delta}\right) + \frac{u_\infty \Delta T}{R}\right] = \alpha\left(\frac{\Delta T}{\delta^2} + \frac{2}{R}\frac{\Delta T}{\delta} + \frac{\Delta T}{R^2} + \frac{\Delta T}{R^2}\right). \tag{101}$$

From the solutions of the energy equation for a growing but stationary bubble, the boundary-layer thickness δ is very small compared to the radius of a bubble; i.e.,

$$\delta \ll R. \tag{102}$$

Retaining only the terms of the order of $1/\delta^2$, we obtain

$$\frac{\partial T}{\partial r} \simeq \frac{\Delta T}{\delta} \simeq \Delta T\left(\frac{1}{\alpha t} + \frac{u_\infty}{\alpha R}\right)^{1/2}. \tag{103}$$

We can therefore assume that the average temperature gradient over the bubble surface must be of the form

$$\overline{\frac{\partial T}{\partial r}}\bigg|_R \simeq (T_\infty - T_{sat})\left(\frac{a}{\alpha t} + \frac{b u_\infty}{\alpha R}\right)^{1/2}, \tag{104}$$

where a and b are some numerical coefficients. Combining Eqs. 87 and 104, we obtain

$$\frac{dR}{dt} = \frac{k}{\rho_v L}\overline{\frac{\partial T}{\partial r}}\bigg|_R = \frac{k(T_\infty - T_{sat})}{\rho_v L}\left(\frac{a}{\alpha t} + \frac{b u_\infty}{\alpha R}\right)^{1/2}. \tag{105}$$

For a stationary bubble growing in a superheated liquid, Plesset and Zwick[8] obtained the asymptotic solution of the energy equation. Comparing their solution

$$\frac{dR}{dt} = \frac{k(T_\infty - T_{sat})}{\rho_v L}\left(\frac{3}{\pi \alpha t}\right)^{1/2} \tag{106}$$

with Eq. 105, we find that

$$a = \frac{3}{\pi}. \tag{107}$$

Levich[31] solved the steady-state diffusion equation for a droplet moving at uniform velocity. Ruckenstein[36] used the solution of Levich for heat transfer between a moving bubble and the surrounding liquid. Ruckenstein's final expression for the average temperature gradient (or concentration gradient for the diffusion equation) is

$$\left.\overline{\frac{\partial T}{\partial r}}\right|_R = (T_\infty - T_{sat})\left(\frac{2}{3\pi}\frac{u_\infty}{\alpha R}\right)^{1/2}. \tag{108}$$

On comparing Eq. 108 with Eq. 104, we obtain

$$b = \frac{2}{3\pi}. \tag{109}$$

Equation 105 for the rate of growth of a moving bubble can now be expressed as

$$\frac{dR}{dt} = \frac{k(T_\infty - T_{sat})}{\rho_v L}\left(\frac{3}{\pi\alpha t} + \frac{2u_\infty}{3\pi\alpha R}\right)^{1/2}. \tag{110}$$

Although this analysis provides a simple relation describing the influence of velocity on bubble growth rate, it does not consider the effect of growth rate on the velocity of a bubble.

D. Present Work

1. Stationary Medium

From the review presented in the previous section, we can see that most investigators have attempted to solve the energy equation for a bubble that is rising at uniform velocity in a uniformly superheated liquid. Pinto and Davis,[41] on the other hand, considered the equation of motion to determine the effect of growth rate on the velocity of a bubble, but they neglected the effect of velocity on growth rate. As far as we know, no attempts have been made to consider both the effect of velocity on growth rate and the effect of growth rate on velocity. This, of course, requires simultaneous solution of coupled equations.

From the order-of-magnitude analysis of Aleksandrov et al.,[44] we note that

$$\frac{dR}{dt} = \frac{k\Delta T}{\rho_v L}\left(\frac{3}{\pi\alpha t} + \frac{2u_\infty}{3\pi R\alpha}\right)^{1/2} \tag{111}$$

or

$$\frac{dR}{dt} = Ja\left(\frac{3\alpha}{\pi t} + \frac{2}{3\pi}\frac{u\alpha}{R}\right)^{1/2} = f(Ja;\ \alpha;\ t;\ u;\ R). \tag{112}$$

If we consider Basset's one-dimensional equation[45] of motion (Eq. 94) and ignore the terms containing ρ_v, we obtain

$$\frac{du}{dt} = 2g - \tfrac{3}{4}C_D\frac{u^2}{R} - \frac{3u}{R}\frac{dR}{dt} = f\left(u, R, \frac{dR}{dt}\right). \tag{113}$$

Equations 112 and 113 are a set of coupled ordinary differential equations for which numerical solutions can be obtained.

We have used a fourth-order Runge-Kutta integration scheme to solve Eqs. 112 and 113. For the drag coefficient in Eq. 113, we used

$$C_D = \frac{216\nu^2}{gR^3} \text{ (Levich[31])} \tag{114}$$

for

$R < 0.07$ cm,

and

$$C_D = \frac{8}{3}\frac{R^2\rho g}{1.82\sigma} \text{ (Peebles and Garber[42])} \tag{115}$$

for $R \geq 0.07$ cm. The following relations were used to obtain the initial values:

$$t_{init} = 0.001 \text{ s}, \tag{116}$$

$$R_{init} = 2Ja\left(\frac{3\alpha t_{init}}{\pi}\right)^{1/2}, \tag{117}$$

and

$$u_{init} = Ja\left(\frac{3\alpha}{\pi t_{init}}\right)^{1/2}. \tag{118}$$

The initial time being very small, the starting values of radius and velocity did not affect the final solution. This can be seen from Table I, in which we have presented the results for two different initial velocities. To check the accuracy of the solution, we made computations with different time intervals. A time increment of 10^{-5} s gave a fairly accurate and stable solution (see Table II).

TABLE I. Results for Water Vapor [Ja = 5.0; p = 1 atm (0.1 MPa)] with Two Values of Initial Velocity

Time, s	Radius, cm		Velocity, cm/s	
	$u_{init} = \frac{dR}{dt}$	$u_{init} = 0$	$u_{init} = \frac{dR}{dt}$	$u_{init} = 0$
0.005	0.0292	0.0288	4.305	3.787
0.1	0.1893	0.1889	25.52	25.50
0.9	0.7500	0.7498	16.99	16.99

TABLE II. Results for Water Vapor [Ja = 5.0; p = 1 atm (0.1 MPa)] with Two Values of Time Increment

Time, s	Radius, cm		Velocity, cm/s	
	$\Delta t = 10^{-5}$ s	$\Delta t = 0.5 \times 10^{-5}$ s	$\Delta t = 10^{-5}$ s	$\Delta t = 0.5 \times 10^{-5}$ s
0.005	0.0292	0.0291	4.305	4.301
0.1	0.1893	0.1893	25.52	25.52
0.9	0.7500	0.74999	16.99	16.99

We have compared our results with experimental measurements of Florschuetz et al.[40] Figures 14-16 show that our predictions are in close agreement with their measurements.

Results have been obtained for water and sodium for a range of Jakob numbers. Figures 17-21 show the effects of Jakob number on velocity and on growth rates. The growth rate and velocities of sodium-vapor bubble are compared (Figs. 21 and 22) with the growth rates and velocities of water-vapor bubble. It can be seen that the rising velocity has a greater influence on the growth rate of a water-vapor bubble than that of a sodium-vapor bubble. This is because, for the same values of Jakob number, the radius of a sodium-vapor bubble is larger than that of a water-vapor bubble. This produces a larger drag force, lowers the velocity, and, consequently, reduces the effect of velocity on the growth rate. Since the velocity is lower for higher Jakob numbers, the influence of velocity on the growth rate decreases with the increase in Jakob numbers. This also can be seen from our results presented in Figs. 17-20 and 22. Figure 23 shows the effect of velocity on the growth of a sodium-vapor bubble.

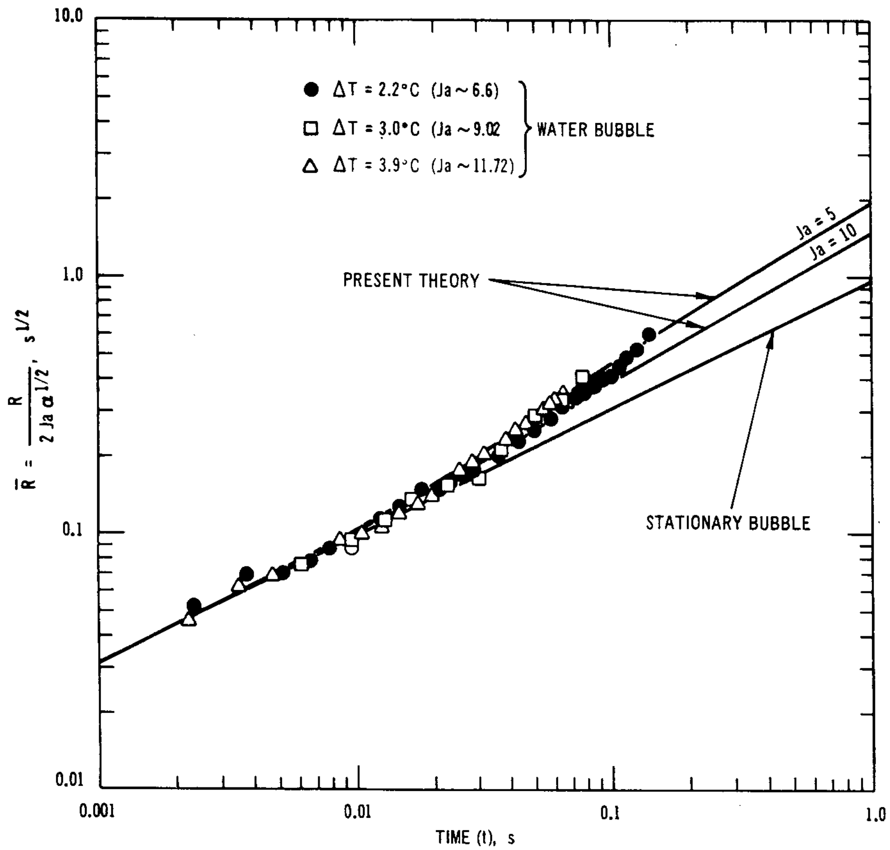

Fig. 14. Comparison of Present Theory with Measurements of Florschuetz et al. for a Water–vapor Bubble

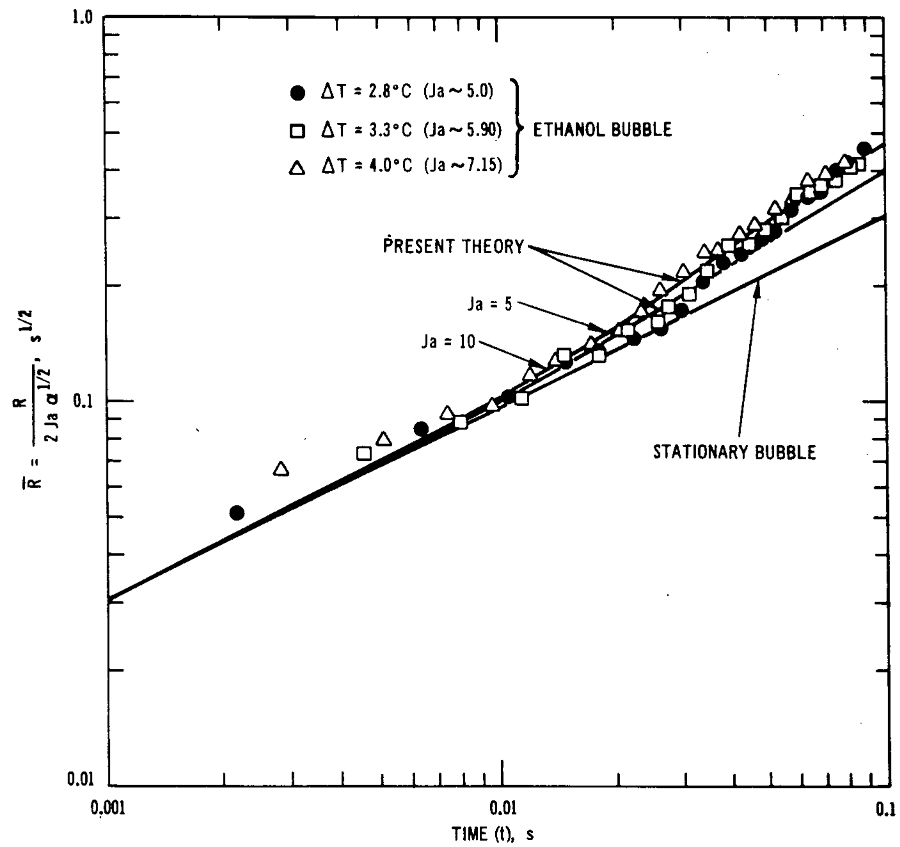

Fig. 15. Comparison of Present Theory with Measurements of Florschuetz et al. for an Ethanol–vapor Bubble

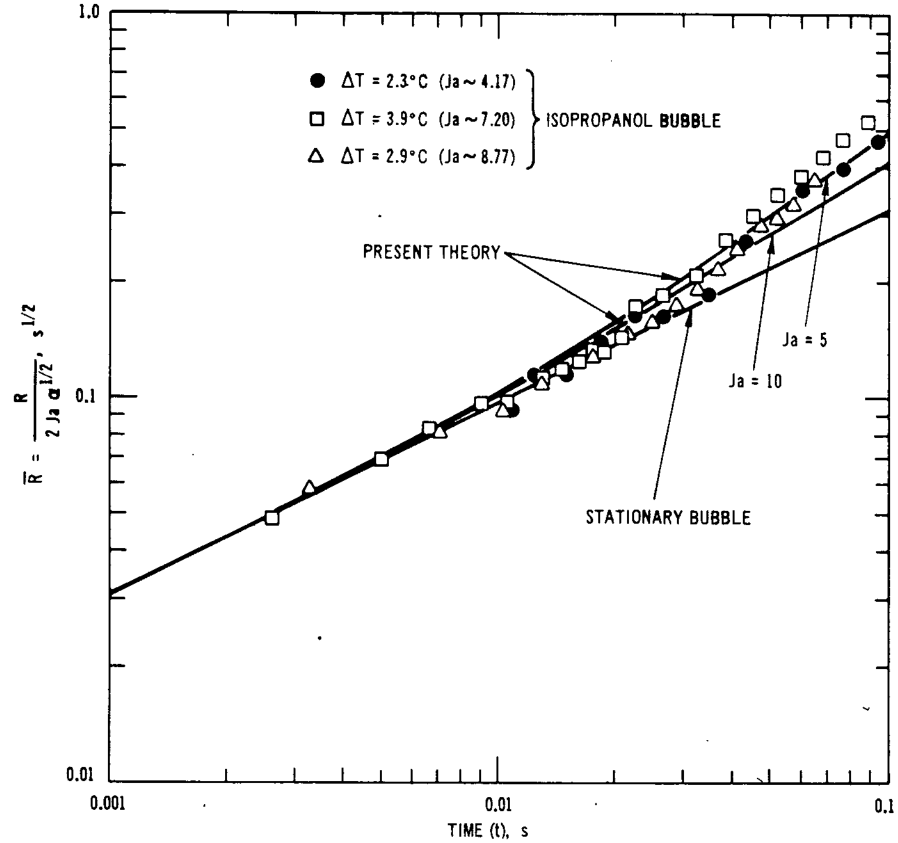

Fig. 16. Comparison of Present Theory with Measurements of Florschuetz et al. for an Isopropanal-vapor Bubble

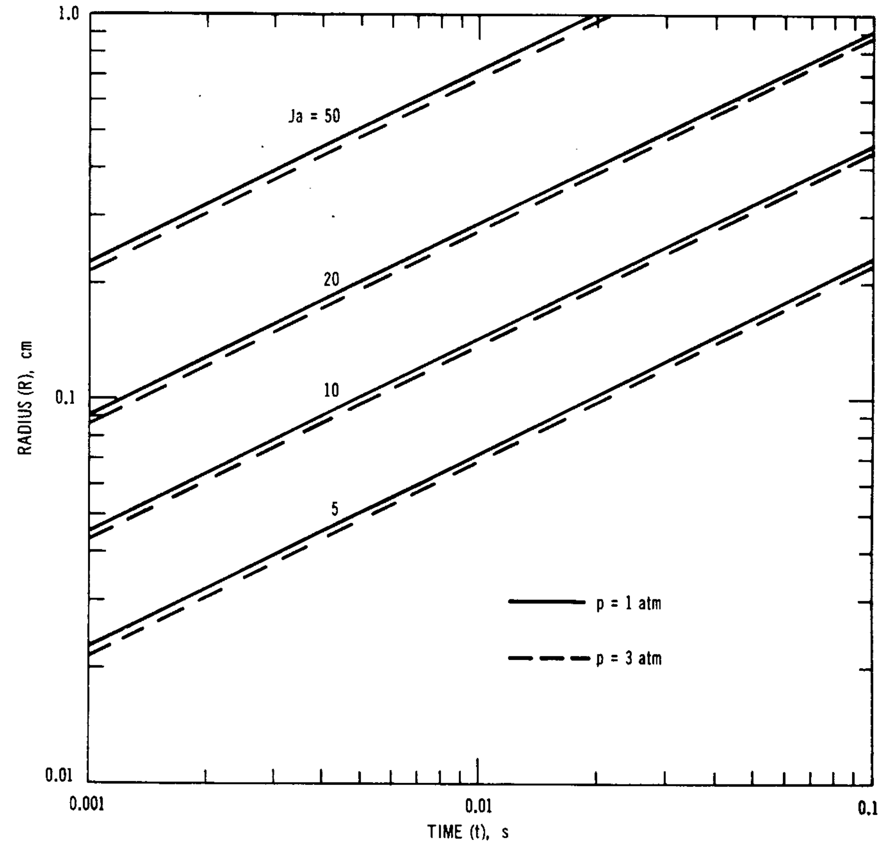

Fig. 17. Effect of Jakob Number on Growth of a Sodium-vapor Bubble. Conversion factor: 1 atm = 0.1 MPa.

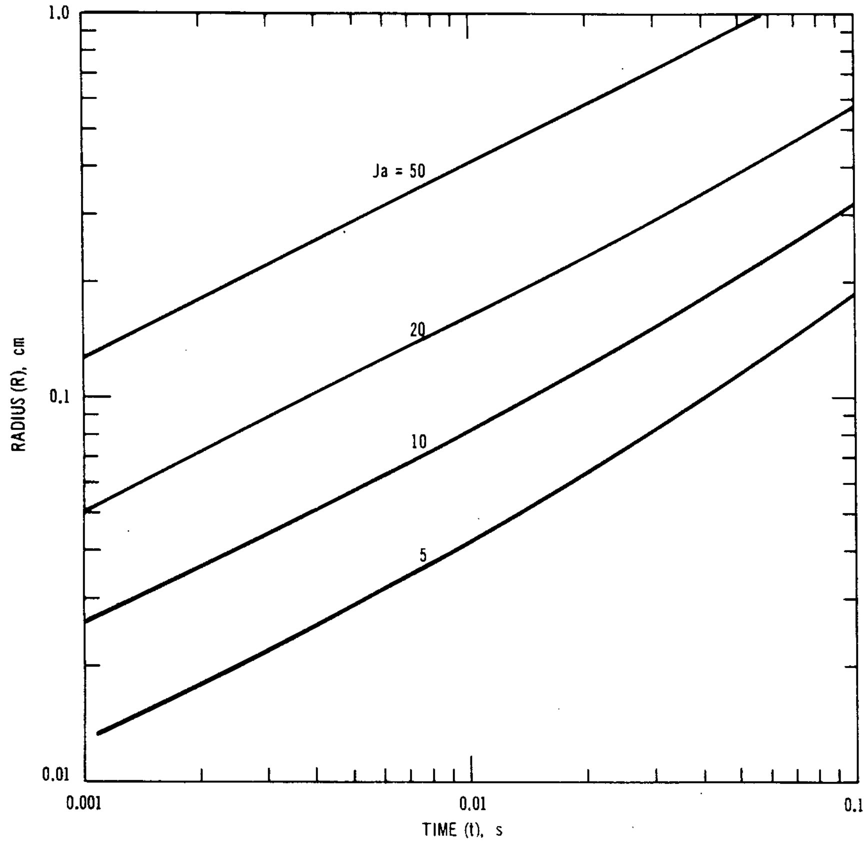

Fig. 18. Effect of Jakob Number on Growth of a Water-vapor Bubble [p = 1 atm (0.1 MPa)]

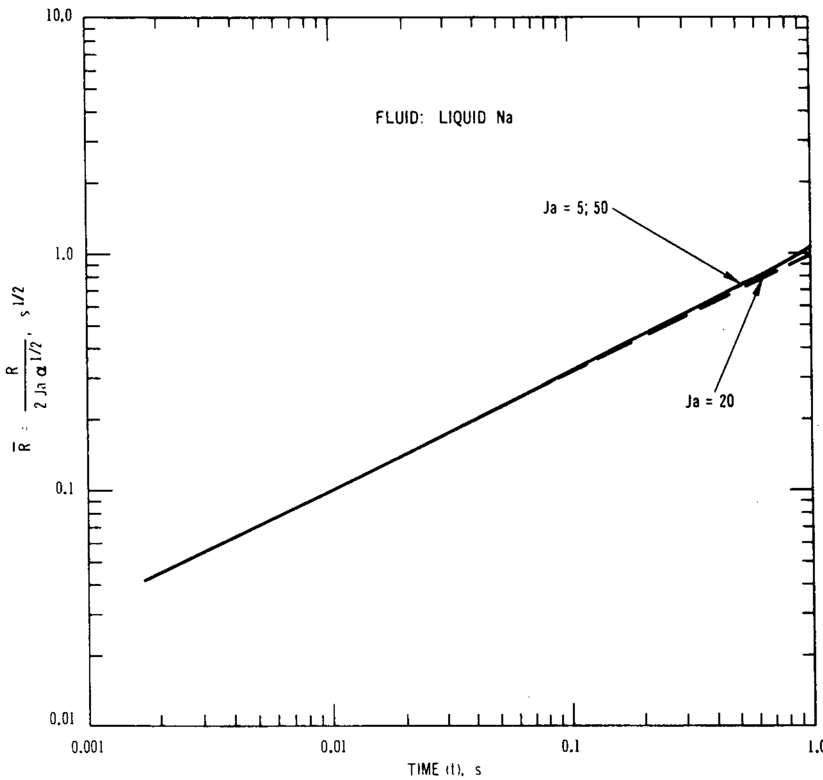

Fig. 19. Effect of Jakob Number on Parameter $\bar{R}$ for a Sodium-vapor Bubble

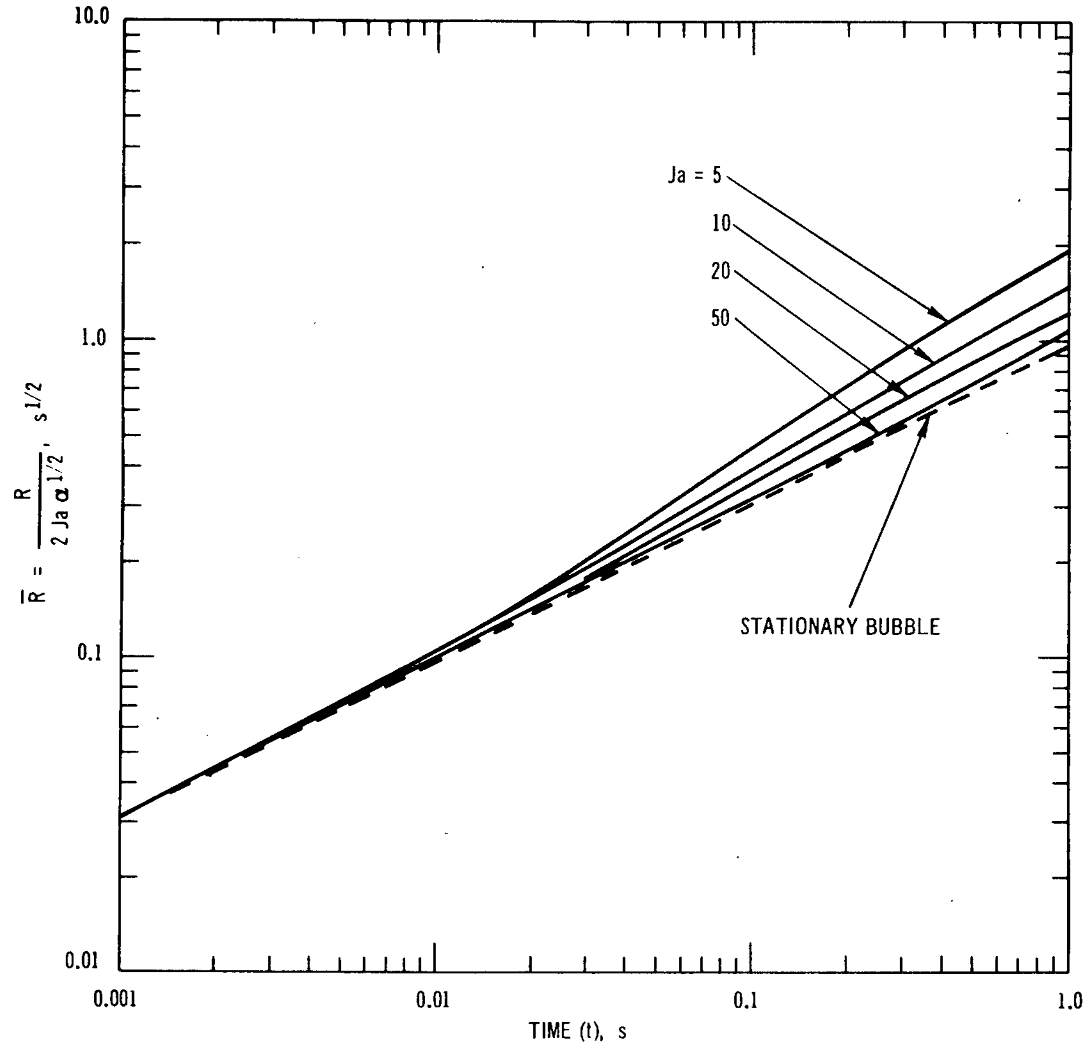

Fig. 20. Effect of Jakob Number on Parameter $\bar{R}$ for a Water-vapor Bubble

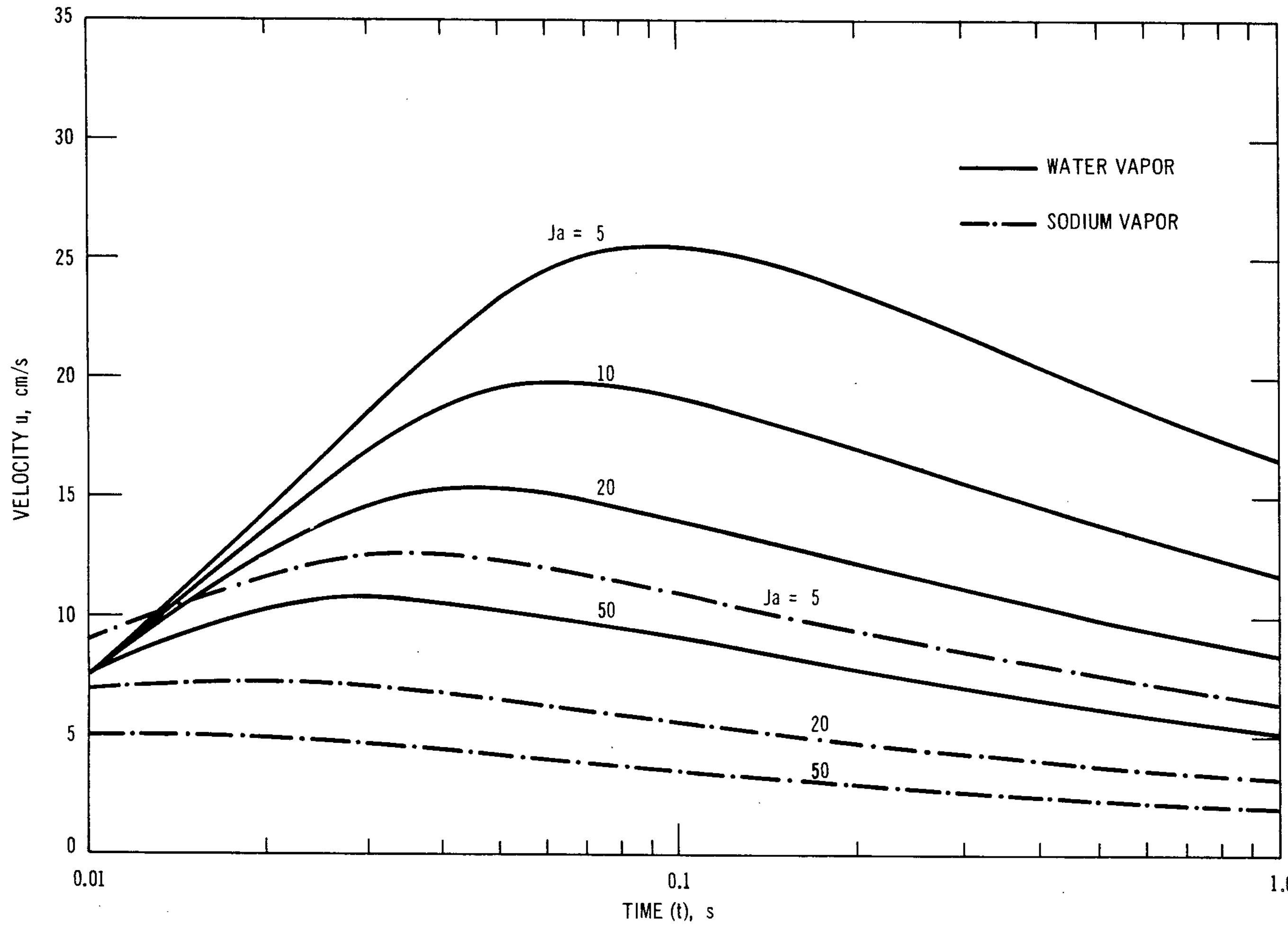

Fig. 21. Effect of Jakob Number on Rising Velocity of a Bubble [p = 1 atm (0.1 MPa)]

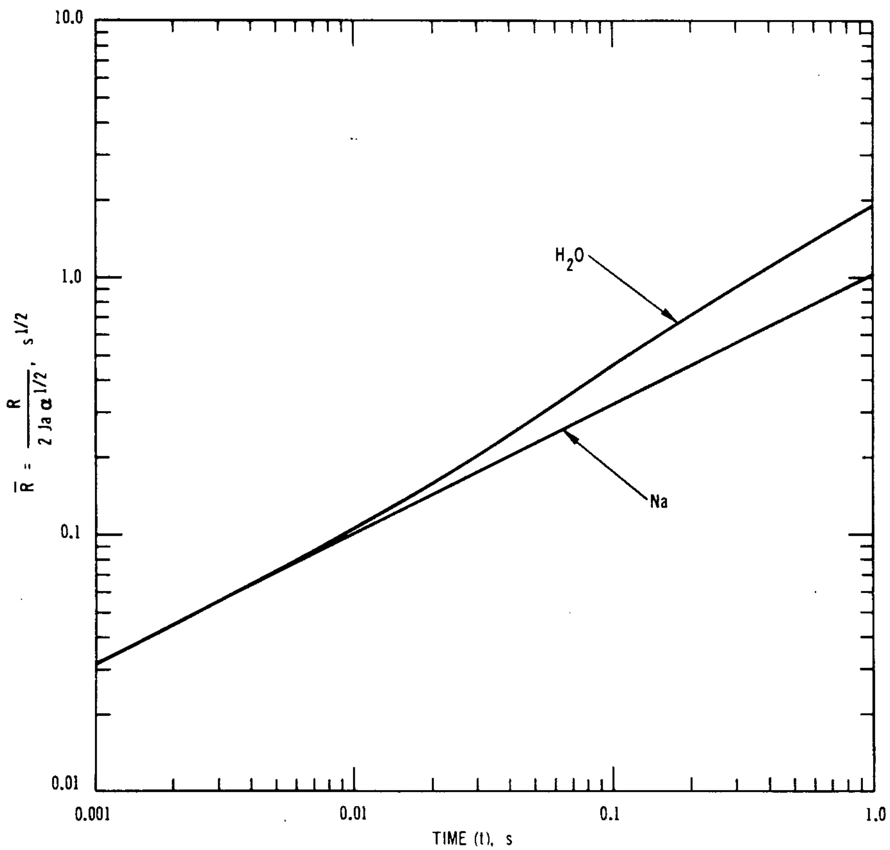

Fig. 22. Growth of a Vapor Bubble in a Superheated Liquid (Ja = 5)

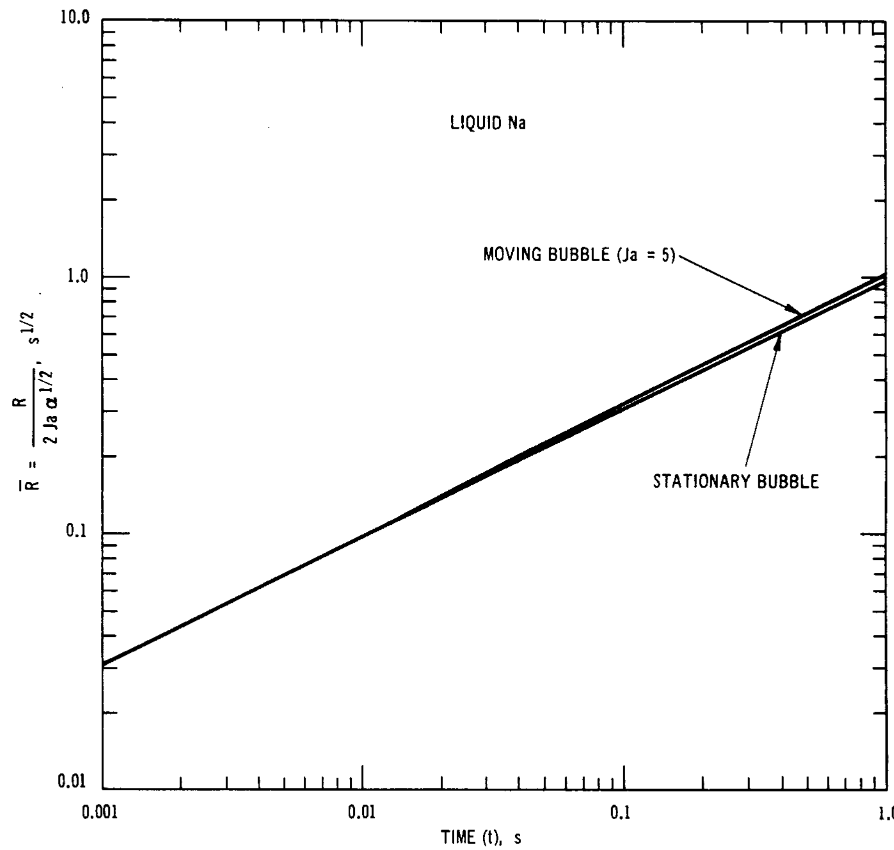

Fig. 23. Effect of Velocity on Growth of a Sodium-vapor Bubble

2. Moving Medium

When a bubble is growing in a moving medium, the surrounding pressure p_∞ may vary significantly along the path of a rising bubble. Although the surrounding pressure does not explicitly enter the equation for growth rate, it influences the growth rate by varying the vapor density.

The variation of surrounding pressure differs from case to case because it depends on the geometry of a channel, flow rate, fluid properties, etc. It is difficult to obtain a generalized solution that is applicable to all flow situations; however, we can present a set of ordinary differential equations that can be solved for a given flow and geometry situation.

Consider a bubble that is rising with a velocity u in a moving medium of velocity u_ℓ. Let the surrounding pressure p_∞ be a known function of the axial distance (height) Z:

$$p_\infty = p_\infty(Z). \tag{119}$$

If we assume that vapor is saturated, we can evaluate vapor density ρ_v at p_∞ from the equation of state; hence,

$$\rho_v = \rho_v(p_\infty). \tag{120}$$

Equations 112 and 113 are still applicable for a moving medium, except that:

(a) We now replace u with the relative velocity

$$u_{rel} = u - u_\ell. \tag{121}$$

(b) Our assumption of uniform Jakob number is no longer valid.

Thus,

$$\frac{dR}{dt} = Ja\left(\frac{3\alpha}{\pi t} + \frac{2}{3\pi}\frac{u_{rel}\,\alpha}{R}\right)^{1/2} = f(Ja(\rho_v); \alpha; t; u_{rel}; R). \tag{122}$$

and

$$\frac{du_{rel}}{dt} = 2g - \tfrac{3}{4}C_D\frac{u_{rel}^2}{R} - \frac{3u_{rel}}{R}\frac{dR}{dt} = f\left(u_{rel}; R; \frac{dR}{dt}\right). \tag{123}$$

The bubble velocity u is related to axial distance (height) Z by

$$\frac{dZ}{dt} = u. \tag{124}$$

Equations 119-124 are a set of coupled equations that can be solved numerically for a given situation.

VI. CONCLUDING REMARKS

1. In this report we have restricted our attention to the growth of a single spherical bubble.

2. From the brief reviews presented here, we can see that considerable work has been done to determine (a) the growth rate of a stationary bubble and (b) the motion of a constant-volume bubble.

3. In a real situation, the growth and motion (rising) of a bubble usually take place concurrently. Due to the complexity of interrelations, it is difficult to obtain exact solutions of coupled differential equations.

4. Very little work has been done to determine the growth of a moving bubble.

5. Experimental measurements have confirmed that the growth of a moving bubble is greater than that of a stationary bubble.

6. We have presented (see Sec. V.D) a set of coupled differential equations to determine the growth and velocity of a moving bubble. Our numerical solution of these equations is in agreement with the experimental data of Florschuetz et al.[40]

7. Results have been obtained for a sodium-vapor bubble and a water-vapor bubble for ranges of pressures and Jakob numbers.

8. Initially, the buoyancy force dominates the motion of a bubble, and the velocity of a bubble increases. As the radius of a bubble increases, the drag force increases and the velocity starts to decrease.

9. We find that, for the same Jakob number,

$$\left(\frac{dR}{dt}\right)_{\text{sodium}} > \left(\frac{dR}{dt}\right)_{\text{water}}$$

   and

$$(\text{velocity})_{\text{sodium}} < (\text{velocity})_{\text{water}}.$$

10. The effect of velocity on the growth of a sodium-vapor bubble is not very large because the velocity of a sodium-vapor bubble is comparatively very low.

11. Although considerable work has been done in bubble dynamics, many phenomena relating to this subject are not understood, e.g., bubble-to-bubble interaction, bubble-to-surface interaction, the effect of nonuniform temperature field on the growth of a bubble, and thermodynamic state at the start of nucleation. Further research in this area will generate answers to some of these questions.

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